RENAISSANCE®

# Star CBM Math
# Technical Manual

RENAISSANCE
Star CBM
Math

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11/2020 Star CBM Math

# Contents

# Introduction

## Star CBM Math: Screening and Progress-Monitoring Assessment

In 2019, under the direction of Dr. Scott McConnell, and with the collaboration of the Psychometrics, Research, Learning Sciences, Engineering, and Content teams at Renaissance, the vision for Star CBM was born. Star CBM was built on over 40 years of research, started by Dr. Stanley Deno at the University of Minnesota. Dr. Deno and his colleagues were committed to developing rigorous, easy-to-use measures of academic achievement and progress that would help teachers better serve their students.

Curriculum-based measurement (CBM) has the dual advantage of being supported by years of research and development and by being steadily improved through use by tens of thousands of teachers and students (Deno, 1985, 1997; Espin, McMaster, Rose, & Wayman, 2012). CBM has been demonstrated to be an important tool for identifying students in need of additional intervention, and in monitoring the effects of that intervention over time. Star CBM builds on both these empirical and practical foundations to provide robust, simple, and technically sophisticated resources for elementary teachers and their students.

In the 2019–2020 school year, Renaissance launched a full field test for Star CBM in Reading and Math from grades K through 3. Participants representing urban and rural, large and small districts from 26 states nationwide participated in the year-long field test that included universal screening and progress monitoring studies. The resulting data was the foundation for user norms and benchmarks for the 2020–2021 school year.

## Star CBM Math Purpose

Star CBM Math is designed to meet and exceed National Center on Intensive Intervention (NCII) standards for universal screening and progress monitoring tools[1], as well as best practices articulated by teachers, researchers, and program leaders for practices like Response to Intervention/Multi-tiered Systems of Support, special education and other compensatory or intensive intervention efforts, and a host of other educational interventions.

1 https://intensiveintervention.org/about-charts-resources

Star CBM data is driven by user norms, contains resulting benchmarks and risk categories, and the user interface contains guidance for teachers to screen seasonally with a single recommended measure for the most efficient assessment experience. Additionally, the Star CBM application prompts teachers to set goals for supplemental intervention for students who are in at-risk or on-watch categories. Star CBM Math is also flexible enough to achieve out-of-grade-level testing as desired/needed.

Although Curriculum-Based Measurement has its early roots with Dr. Stan Deno and the University of Minnesota in reading, mathematics, spelling, and written expression, the CBM field of measures has grown over the years. As a longtime colleague of Dr. Deno, Dr. McConnell focused on the principles of General Outcome Measurement (Fuchs & Deno, 1994) to identify a series of measures that would describe early elementary students' largest stepping stones on the path to mathematical proficiency. The journey for Star CBM Math begins with numeral recognition and comparison of single quantities, progresses to basic addition and subtraction, and finally mixed addition and subtraction and simple multiplication. These measures emphasize number sense and basic operations as foundational to further mathematical skill and competence.

The Star CBM Math test's repeatability and flexible administration options provide specific advantages for everyone responsible for the education process:

- For students, Star CBM Math provides a brief, challenging test experience that is friendly and fun.
- For teachers, Star CBM Math provides the opportunity to observe each child one at a time, and also helps to facilitate individualized instruction by identifying children who need intervention or enrichment most and helping teachers to analyze their progress.
- For administrators, Star CBM Math provides a window into each individual classroom to view accurate reports on performance on universal screening and progress monitoring and improve building or district-level decision-making.

This manual documents the suitability of Star CBM Math testing for these purposes and demonstrates quantitatively how well this innovative instrument in math assessment performs.

# Design of Star CBM Math

Star CBM Math rests on great foundations—specifically, the work of Dr. Stanley Deno, the originator of this approach to educational assessment (Deno, 1985, 1997; Deno & Mirkin, 1977; Espin et al., 2012).

As Deno (2006) described it, "CBM exists as one technically adequate approach for taking a more functional problem-solving approach to the prevention and solution of educational problems. Evidence exists that professional educators can increase their problem-solving effectiveness through the use of progress monitoring of student development and by systematically responding to those data as they reflect student growth."

Dr. Deno and his colleagues' core principles also guided development of grade-level math measures included in Star CBM Math. Fundamentally, every element in this new set of measures hews closely to his focus on rigor and simplicity. We designed for rigor by producing measures that can be trusted and are meaningful, meeting research-based standards for test reliability and validity. Just as important, we designed for simplicity in administration, scoring, interpretation, and use of each measure. In other words, we designed tools that teachers can use, quickly and efficiently, to assess their students' achievement, make key instructional decisions, and monitor the effects of their decisions over time.

Star CBM Math is indeed a General Outcome Measure (GOM); while the content of assessment varies across seasons and grades, each measure is intended to represent a child's growth in the general domain of math. In this way, our approach to assessment across the grades reflects the concept of "heterotypic development"—competence in a general construct represented by improving performance in a specific area that contributes to the greater whole. In reading, this can be seen in Scarborough's "reading rope" (Scarborough, 2001), where different strands of successful reading (e.g., letter-sound correspondence, phonological awareness, word reading) develop in concert as the child's achievement progresses. In Star CBM Math, these different aspects of mathematical achievement are represented initially by number sense, then quantity comparison, and then arithmetic operations. As a result, strands (and features of strands) that are learned earlier can be used for assessing younger children, with measures increasing in complexity or challenge as children's achievement progresses.

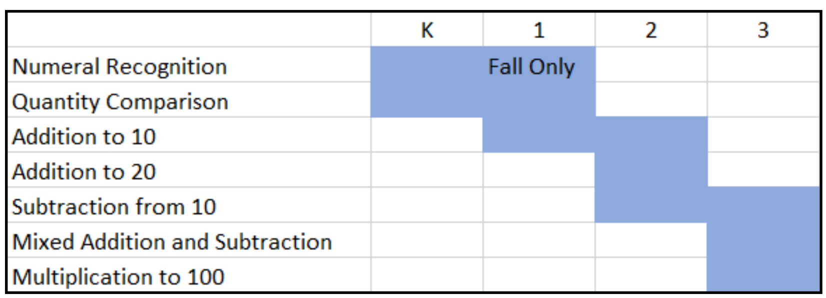
## Initial Set of Measures for Star CBM Math

The developmental progression of components of math can be seen in the array of measures included in Star CBM Math. We have selected and designed measures to be administered from the first days of kindergarten, when children are very early in their learning of the complex skills of math, to the end of third grade, when we expect students to be mastering more complex operations and understanding of mathematical content. That progression is clear in measures themselves and in the scores observed in the field test, across seasons and grades.

**Table 1: Screening Recommendations and Grade-Level Norms**

| Grade | Screening Recommendation | Grade-Level Norms Available |
|---|---|---|
| K | Numeral Recognition | • Numeral Recognition<br>• Quantity Comparison |
| 1 | Quantity Comparison | • Quantity Comparison<br>• Addition to 10<br>• Numeral Recognition (Fall only) |
| 2 | Addition to 20 | • Addition to 20<br>• Subtraction from 10<br>• Addition to 10 |
| 3 | Mixed Addition and Subtraction | • Mixed Addition and Subtraction<br>• Subtraction from 10<br>• Multiplication to 100 |

**Figure 1: Grade-Level Norms Available**




These Star CBM Math measures are part of a larger set of Curriculum-Based Measures, including Star CBM Reading and two Rapid Automatic Naming measures for screening for characteristics associated with dyslexia. Star CBM Math and Star CBM Reading (with Rapid Automatic Naming) can be used alone or in any combination.

## Star CBM Field Test

To evaluate the rigor and usefulness of Star CBM, we designed a field test for the 2019–2020 academic year where each measure would be administered within and across grades. Our goal was to identify a subset of these measures that provided useful and direct information on students' progress toward proficiency in each domain. Our goal in the field test was to identify measures that a) yielded useful data on student performance across at least one academic year; b) correlated with direct measures and accepted predictors of proficiency in each domain; c) related, conceptually and empirically, to other measures in the suite (those used for younger or older students); and d) proved useful to assess student progress over relatively short (e.g., weekly) intervals.

Two separate studies were initiated. The first, Seasonal Screening, focused on psychometric characteristics of Star CBM when used for periodic assessment of every student in a class. In this study we evaluated measure reliability, validity, and "classification accuracy," or performance against standards for assessing candidacy for supplemental intervention (or "risk"). Participating teachers were asked to assess all or virtually all students in their class on 4 occasions: Fall (October), 2 times in Winter (January, and 2 weeks later), and Spring (May). The second study, Progress Monitoring, asked teachers to conduct repeated assessment of select students' performance 11 times over a period of approximately 22 weeks (every other week). In addition to adding information to the Seasonal Screening study, this progress monitoring study was designed to evaluate Star CBM measures' sensitivity to growth over time—a critical feature of measures used to evaluate the efficacy of Tier 2 and Tier 3 interventions.

Completion of the Star CBM field test was directly affected by COVID-19 school closures, reducing both the number of participants completing Spring seasonal screening measures and the number of students for whom we had sufficient progress monitoring data to complete all of our planned analyses. However, more than 3,000 children participated in Fall and Winter Screening or some portion of planned Progress Monitoring assessments, and a small number of students were assessed in Spring. While evaluation of Star CBM will continue in future years, sufficient information was available from the field test to both demonstrate psychometric qualities of Star CBM, and to offer useful guidance for programs and teachers using these measures.

# Overarching Design Considerations

A summary design statement of Star CBM is "freedom with guardrails." Throughout the product conception, design, and development, we have worked to provide guardrails to teachers who may be looking for our recommendations and insights, but also to provide multiple assessment options to allow for dynamic choice based on student needs, teacher needs, school and district requirements, limitations of resources, time, and a myriad of other realities faced in schools across the country on a daily basis. And out of that understanding and empathy, Star CBM was born.

We also worked to achieve two goals we believe are critically important to teachers and students: simplicity and rigor. Rather than providing a large variety of scores, graphs, and analyses, we continue to work toward direct, simple information that helps teachers and their students assess current performance, determine any need for additional intervention, and monitor the effects of that additional intervention when provided. Additionally, we wanted the information drawn from Star CBM to be trustworthy and useful—to meet the highest standards of rigor for educational testing of this sort. Together, simple and rigorous measures and guidance with guardrails are defining features of Star CBM.

An important consequence of these fundamental Star CBM Math design decisions leads to giving teachers a range of choices, at the point of administration for each student, for each test session. Teachers are able to administer Online, on Paper, or in a Mixed format in which the student is viewing a paper form while the teacher scores online. These options allow for strengths of different administration scenarios to be leveraged according to what is most important to the teacher and student at that time, while still remaining psychometrically rigorous, with forms consistent across the three options and guiding testing best practices for administration via scripting, reminders, and consistency in the user interface to maintain fidelity.

Additionally, Star CBM Math is designed to be the most efficient with student and teacher time; a single screening measure is suggested for each grade, allowing the teacher to gather information that can be directly analyzed for current performance as well as growth across the entire year. Teachers can decide to use only the recommended screening measure, or they can assess with additional measures as well. They may also choose to progress monitor depending on student learning gaps or with the goal of challenging students who are already at/above benchmark.

Teachers are also provided normative information to evaluate student performance, principally to help teachers identify those students who may require additional or different intervention, and for whom they should consider setting a goal and monitoring progress toward that goal. Teachers are provided information to set goals appropriate for individual children, including moderate, ambitious, and custom goals and options for setting the end date for reaching a student's goal.

## Test Interface

The Star CBM Math test interface was designed to be both simple and effective. Students will typically verbalize their answers to individual items, and teachers score child performance based on those responses. Each Star CBM Math measure is one minute in length. Further, teachers can administer the test one of three ways, including an Online format where both the teacher and student work with computers or other digital devices, a Print format where the student reads from a printed page and the teacher scores on a teacher version of that form, and a Mixed format where the student reads from paper and the teacher scores on a digital device. For details, see the Star CBM Test Administration Manual.

## Test Length

All Star CBM Math measures are one minute in length. Brief demonstration or instruction is provided to the student, adding 30–45 seconds to administration. In this way, if a teacher is administering the single recommended screening measure per grade, an estimate of the time needed to screen the entire class can be made by multiplying the number of students in the class by 3 minutes—one minute to read directions, one minute to test, and one minute to transition between students. In all three types of administrations, the scoring is completed in real time during the test itself. However, in Print administrations, that scoring record must later be added into the software, which may take another minute. The Online and Mixed administration formats are the most time efficient.

## Test Repetition

Star CBM Math score data can be used for multiple purposes, such as screening, placement, planning instruction, benchmarking, progress monitoring, and outcomes measurement. The frequency with which the assessment is administered depends on the purpose for assessment and how the data will be used. Renaissance Learning recommends assessing students only as frequently as necessary to get the data needed.

Star CBM Math was designed for seasonal screening purposes three times a year. Additionally, teachers who want to monitor student progress more closely or use the data for instructional planning may use it more frequently. Star CBM Math may be administered as often as every week for progress monitoring purposes, but many teachers will find every-other-week intervals more practical. Star CBM Math keeps track of the forms presented to each student and will not present the same form to a student until he or she has seen all 20 in random order, then will begin a second pass through the 20 forms, again in a randomly assigned order.

# Test Security

Star CBM Math includes a number of security features designed primarily to keep students' records confidential, while allowing teachers and others to share results as needed.

## User Roles and Permissions

When teachers administer Star CBM Math assessments to students in the Mixed or Print formats, students do not log in to the software. When students taking Online assessments log into Star CBM Math, they do not have access to the same functions that teachers, administrators, and other personnel can access. Once teachers grant access, students are allowed to take the test, but no other features in Star CBM Math are available to them; therefore, they have no access to confidential information.

## Individualized Tests

Every Star CBM Math measure is randomly assigned from one of 20 forms for each measure, algorithmically selected at the point of clicking on the Start or Print Assessment button. The application keeps track of which forms each student has already seen, and they will see all 20 before circling back to any form for a second time.

## Access Levels and Capabilities

Each personnel user's level of access to a Renaissance program depends on the primary position assigned to that user. Each primary position is part of a user permission group. There are six of these groups: district level administrator, district dashboard owner, district staff, school level administrator, school staff, and teacher. By default, each user permission group is granted a specific set of user permissions; each user permission corresponds to one or more tasks that can be performed in the program. The user permissions for these groups can be changed, and user permissions can be granted or removed on an individual level.

The security of the Star CBM Math data is also protected by each person's user name (which must be unique) and password. User names and passwords identify users, and the program only allows them access to the data and features that they are allowed based on their primary position and the user permissions that they have been granted. Personnel who log in to Renaissance (teachers, administrators, or staff) must enter a user name and password before they can access the data and create reports. Without an appropriate user name and password, personnel cannot use the Star CBM Math application.

### Password Entry

When teachers administer assessments using the Online format, students are required to enter a user name and password to log in. Teachers also start Online assessments for specific students, so other students who log in do not see the assessments. This helps to ensure that students cannot take assessments using other students' names.

Students do not log in for assessments in Mixed or Print format.

### Disclaimer

While Renaissance can do much to provide specific measures of test security, the most important line of defense against unauthorized access or misuse of the program is the user's responsibility. Teachers and test monitors need to be careful not to leave their session active and unattended, and to monitor all testing to prevent students from practicing content on forms before an assessment session begins. Taking these simple precautionary steps will help maintain Star CBM Math's security and the quality and validity of its scores.

## Test Administration Procedures

In order to ensure consistency and comparability of results to the Star CBM Math norms, teachers administering Star CBM Math tests should follow standard administration procedures as described in the Test Administration

Manual. The testing environment should be as free from distractions for the student as possible.

The Test Administration Manual included with the Star CBM Math product describes the standard test orientation procedures that teachers should follow to prepare their students for the Star CBM Math assessments; teachers will also find instructions for the specific measures as they administer assessments. These instructions are intended for use with students of varying ages; however, Star CBM Math was designed to be administered to students in grades K–3, the grades for which normative data is available. The measure instructions were successfully field-tested with students ranging from grades K–3. It is important to use these same instructions with all students before they take the Star CBM Math measures.

# Star CBM Math Content Specification

Star CBM Math is a Renaissance product that allows teachers to directly administer quick, one-minute assessments to students in a one-on-one setting targeting key building blocks in math. Teachers may administer the content to students in one of three formats: Online, Print, or Mixed format. Each Online, Print, and Mixed format form contains the same placement of items, providing a consistent assessment experience for every student. Star CBM Math consists of 140 operational test forms aligned to seven measures at Grades Kindergarten through 3. There are 20 forms for each measure at each grade.

The measures for which content was created and the grades of students evaluated in the field test follow:

- Numeral Recognition (Grades K, 1)
- Quantity Comparison (Grades K, 1)
- Addition to 10 (Grades K, 1, 2)
- Addition to 20 (Grades 1, 2)
- Subtraction from 10 (Grades 2, 3)
- Mixed Addition and Subtraction (Grades 2, 3)
- Multiplication to 100 (Grade 3)

The Renaissance Content team created student-facing forms used for each measure. Digital PDFs and printed teacher and student manuals were developed by Content teams and used for a CBM field test. The field test, utilized in the 2019–2020 school year, yielded a successful 100,000+ administrations from which robust psychometric data was collected.

In 2020, Renaissance redelivered Star CBM Math forms for both online and paper-based delivery to provide teachers with choice in administration formats.

# Assessment Blueprint Characteristics

The Star CBM Math assessments consist of items grouped into forms that each target one of seven measures. The number of items on each form vary pending the measure being assessed. Table 2 shows a breakdown of the number of items per form per measure for each respective grade.

**Table 2: Number of Forms and Items for Each Star CBM Math Measure by Grade**

| Domain | Measure | Number of Forms | Items Per Form | Grades Covered |
|---|---|---|---|---|
| Numeracy and Mathematics | Numeral Recognition | 20 | 80 | K–1 |
| | Quantity Comparison | 20 | 60 | K–1 |
| | Addition to 10 | 20 | 80 | K–2 |
| | Addition to 20 | 20 | 80 | 1–2 |
| | Subtraction from 10 | 20 | 80 | 2–3 |
| | Mixed Addition and Subtraction | 20 | 80 | 2–3 |
| | Multiplication to 100 | 20 | 80 | 3 |

# The Star CBM Math Set of Measures

A summary of the content guidelines within each Star CBM Math measure appears in Table 3.

## Hierarchy of CBM Measures

**Table 3: Math Blueprint Content Guidelines Per Measure by Grade**

| Domain | Measure | Blueprint Content Guidelines | Grades Covered |
|---|---|---|---|
| Numeracy and Mathematics | Numeral Recognition | One-minute assessment, numerals 1–100 | K–1 |
| | Quantity Comparison | One-minute assessment, comparing numbers 0–30 | K–1 |
| | Addition to 10 | One-minute assessment, columnar addition problems, with addends 0–10 and sums up to 10 | K–2 |
| | Addition to 20 | One-minute assessment, columnar addition problems, with addends 0–20 in columnar addition, with sums up to 20 | 1–2 |
| | Subtraction from 10 | One-minute assessment, columnar subtraction problems, with minuends varying from 0–10 | 2–3 |
| | Mixed Addition and Subtraction | One-minute assessment, columnar mixed addition and subtraction problems, with addition problem sums to 20 and subtraction problem minuends to 20 | 2–3 |
| | Multiplication to 100 | One-minute assessment, columnar multiplication problems, with factors 0–10 and products no greater than 100 | 3 |

# Form Design Guidelines

The forms for each measure were designed to meet the specifications indicated below.

## Numeral Recognition

### Brief Description

- One-minute assessment of student production of "names" for numerals 0 to 100.
- A population of numerals 0 to 100 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the numerals on each form to ensure the content specifications and exceptions below were met for the measure.

### Specifications for Form Content

- Random sequence of 80 numerals, 0–100
- 16 out of 80 numerals 11–100
- Approximately 64 out of 80 numerals 0–10
- Repetition of numerals 11–100 not permitted in individual probe
- 80 numerals per form

### Exceptions

1. Sequences of more than two numerals that fall in numeric order (e.g., 7-8-9 or 83-84-85) or reverse order (e.g., 9-8-7 or 85-84-83) are avoided.
2. The same number appearing in the form in consecutive positions is avoided.

## Quantity Comparison

### Brief Description

- One-minute assessment where student identifies the larger number in each presented pair, comparing numbers 0 to 30.
- A population of numerals 0 to 30 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the numerals on each form to ensure the content specifications and exceptions below were met for the measure.

### Specifications for Form Content

- Randomly sequenced pairs of random numbers (0 to 30).
- Number pairs where neither number exceeds 10 should occur twice as often, and half of the number pairs should differ by 10 or less.
- Order of presentation, larger and smaller numbers in pairs, randomized across items.
- 60 pairs per form.
- On each form, there are about 40 item pairs (out of 60 total) in which neither number exceeds 10.
- The remaining item pairs (approximately 20) have numbers that may be more than 10.

### Exceptions

1. Sequences of more than two pairs where the correct response is in the same position (e.g., first or second number in pair) are avoided.
2. The same comparison appearing in the form (e.g., have two items that are 5 and 1 in that order) is avoided.
3. Pairs where both numbers are equal are avoided.
4. More than two comparisons in succession, where the correct response is in the same right or left position (e.g., 7 & 9, 2 & 8, 5 & 10), are avoided.

## Addition to 10

### Brief Description

- One-minute assessment where the student reports sums for columnar addition problems with addends 0 to 10 and sums up to 10.
- A population of the sixty-six unique sums found in Table 4 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the sums onto each form to ensure the content specifications and exceptions below were met for the measure.

**Table 4: Unique Sums for Addition to 10 with Addends 0 to 10 and Sums Up to 10 (66 Unique Sums)**

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 + 0 | 0 + 1 | 0 + 2 | 0 + 3 | 0 + 4 | 0 + 5 | 0 + 6 | 0 + 7 | 0 + 8 | 0 + 9 | 0 + 10 |
| 1 | 1 + 0 | 1 + 1 | 1 + 2 | 1 + 3 | 1 + 4 | 1 + 5 | 1 + 6 | 1 + 7 | 1 + 8 | 1 + 9 | – |
| 2 | 2 + 0 | 2 + 1 | 2 + 2 | 2 + 3 | 2 + 4 | 2 + 5 | 2 + 6 | 2 + 7 | 2 + 8 | – | – |
| 3 | 3 + 0 | 3 + 1 | 3 + 2 | 3 + 3 | 3 + 4 | 3 + 5 | 3 + 6 | 3 + 7 | – | – | – |
| 4 | 4 + 0 | 4 + 1 | 4 + 2 | 4 + 3 | 4 + 4 | 4 + 5 | 4 + 6 | – | – | – | – |
| 5 | 5 + 0 | 5 + 1 | 5 + 2 | 5 + 3 | 5 + 4 | 5 + 5 | – | – | – | – | – |
| 6 | 6 + 0 | 6 + 1 | 6 + 2 | 6 + 3 | 6 + 4 | – | – | – | – | – | – |
| 7 | 7 + 0 | 7 + 1 | 7 + 2 | 7 + 3 | – | – | – | – | – | – | – |
| 8 | 8 + 0 | 8 + 1 | 8 + 2 | – | – | – | – | – | – | – | – |
| 9 | 9 + 0 | 9 + 1 | – | – | – | – | – | – | – | – | – |
| 10 | 10 + 0 | – | – | – | – | – | – | – | – | – | – |

### Specifications for Form Content

- Random selection and sequencing, with replacement, of single-digit addends in columnar addition, with sums up to 10; in other words, digits 0 to 10 will occur randomly as top or bottom addend in columnar addition problems that sum to 10 or less.
- “Associative” replications (e.g., 3 + 2 = and 2 + 3 =) may occur in the same form.
- 80 items per form.

### Exceptions

1. All possible combinations of 66 sums, with 14 repetitions are used.
2. The same item appearing in the same form more than twice is avoided.
3. Associative replications adjacent in sequence are avoided.
4. Sequences of more than 2 items where one addend is repeated are avoided.
5. No repeated top addend: 1 + 2, 1 + 5, 1 + 7.
6. No repeated bottom addend: 2 + 8, 0 + 8, 1 + 8.
7. Formula repetition on the same line is avoided.

## Addition to 20

### Brief Description

- One-minute assessment where the student reports sums for columnar addition problems with addends 0 to 20 and sums up to 20.
- A population of the 231 unique sums found in Table 5 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the sums onto each form to ensure the content specifications and exceptions below were met for the measure.

**Table 5: Unique Sums for Addition to 20 with Addends 0 to 20 and Sums Up to 20 (231 Unique Sums)**

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 + 0 | 0 + 1 | 0 + 2 | 0 + 3 | 0 + 4 | 0 + 5 | 0 + 6 | 0 + 7 | 0 + 8 | 0 + 9 | 0 + 10 | 0 + 11 | 0 + 12 | 0 + 13 | 0 + 14 | 0 + 15 | 0 + 16 | 0 + 17 | 0 + 18 | 0 + 19 | 0 + 20 |
| 1 | 1 + 0 | 1 + 1 | 1 + 2 | 1 + 3 | 1 + 4 | 1 + 5 | 1 + 6 | 1 + 7 | 1 + 8 | 1 + 9 | 1 + 10 | 1 + 11 | 1 + 12 | 1 + 13 | 1 + 14 | 1 + 15 | 1 + 16 | 1 + 17 | 1 + 18 | 1 + 19 | – |
| 2 | 2 + 0 | 2 + 1 | 2 + 2 | 2 + 3 | 2 + 4 | 2 + 5 | 2 + 6 | 2 + 7 | 2 + 8 | 2 + 9 | 2 + 10 | 2 + 11 | 2 + 12 | 2 + 13 | 2 + 14 | 2 + 15 | 2 + 16 | 2 + 17 | 2 + 18 | – | – |
| 3 | 3 + 0 | 3 + 1 | 3 + 2 | 3 + 3 | 3 + 4 | 3 + 5 | 3 + 6 | 3 + 7 | 3 + 8 | 3 + 9 | 3 + 10 | 3 + 11 | 3 + 12 | 3 + 13 | 3 + 14 | 3 + 15 | 3 + 16 | 3 + 17 | – | – | – |
| 4 | 4 + 0 | 4 + 1 | 4 + 2 | 4 + 3 | 4 + 4 | 4 + 5 | 4 + 6 | 4 + 7 | 4 + 8 | 4 + 9 | 4 + 10 | 4 + 11 | 4 + 12 | 4 + 13 | 4 + 14 | 4 + 15 | 4 + 16 | – | – | – | – |
| 5 | 5 + 0 | 5 + 1 | 5 + 2 | 5 + 3 | 5 + 4 | 5 + 5 | 5 + 6 | 5 + 7 | 5 + 8 | 5 + 9 | 5 + 10 | 5 + 11 | 5 + 12 | 5 + 13 | 5 + 14 | 5 + 15 | – | – | – | – | – |
| 6 | 6 + 0 | 6 + 1 | 6 + 2 | 6 + 3 | 6 + 4 | 6 + 5 | 6 + 6 | 6 + 7 | 6 + 8 | 6 + 9 | 6 + 10 | 6 + 11 | 6 + 12 | 6 + 13 | 6 + 14 | – | – | – | – | – | – |
| 7 | 7 + 0 | 7 + 1 | 7 + 2 | 7 + 3 | 7 + 4 | 7 + 5 | 7 + 6 | 7 + 7 | 7 + 8 | 7 + 9 | 7 + 10 | 7 + 11 | 7 + 12 | 7 + 13 | – | – | – | – | – | – | – |
| 8 | 8 + 0 | 8 + 1 | 8 + 2 | 8 + 3 | 8 + 4 | 8 + 5 | 8 + 6 | 8 + 7 | 8 + 8 | 8 + 9 | 8 + 10 | 8 + 11 | 8 + 12 | – | – | – | – | – | – | – | – |
| 9 | 9 + 0 | 9 + 1 | 9 + 2 | 9 + 3 | 9 + 4 | 9 + 5 | 9 + 6 | 9 + 7 | 9 + 8 | 9 + 9 | 9 + 10 | 9 + 11 | – | – | – | – | – | – | – | – | – |
| 10 | 10 + 0 | 10 + 1 | 10 + 2 | 10 + 3 | 10 + 4 | 10 + 5 | 10 + 6 | 10 + 7 | 10 + 8 | 10 + 9 | 10 + 10 | – | – | – | – | – | – | – | – | – | – |
| 11 | 11 + 0 | 11 + 1 | 11 + 2 | 11 + 3 | 11 + 4 | 11 + 5 | 11 + 6 | 11 + 7 | 11 + 8 | 11 + 9 | – | – | – | – | – | – | – | – | – | – | – |
| 12 | 12 + 0 | 12 + 1 | 12 + 2 | 12 + 3 | 12 + 4 | 12 + 5 | 12 + 6 | 12 + 7 | 12 + 8 | – | – | – | – | – | – | – | – | – | – | – | – |
| 13 | 13 + 0 | 13 + 1 | 13 + 2 | 13 + 3 | 13 + 4 | 13 + 5 | 13 + 6 | 13 + 7 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 14 | 14 + 0 | 14 + 1 | 14 + 2 | 14 + 3 | 14 + 4 | 14 + 5 | 14 + 6 | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 15 | 15 + 0 | 15 + 1 | 15 + 2 | 15 + 3 | 15 + 4 | 15 + 5 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 16 | 16 + 0 | 16 + 1 | 16 + 2 | 16 + 3 | 16 + 4 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 17 | 17 + 0 | 17 + 1 | 17 + 2 | 17 + 3 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 18 | 18 + 0 | 18 + 1 | 18 + 2 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 19 | 19 + 0 | 19 + 1 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 20 | 20 + 0 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |

### Specifications for Form Content

- Random selection and sequencing, with replacement, of addends 0–20 in columnar addition, with sums up to 20; in other words, digits 0 to 20 will occur randomly as top or bottom addend in columnar addition problems that sum to 20 or less.
- "Associative" replications (e.g., 15 + 4 = and 4 + 15 =) may occur in the same form.
- 80 items per form.

#### Exceptions

1. Associative replications adjacent in sequence are avoided.
2. Sequences of more than 2 items where one addend is repeated are avoided.
   a. No repeated top addend: 1 + 19, 1 + 5, 1 + 10.
   b. No repeated bottom addend: 2 + 18, 0 + 18, 1 + 18.
3. The same item appearing in the same form is avoided (e.g., 3 + 2 = and 3 + 2 = on the same form; there should be 80 unique items per form).

## Subtraction from 10

#### Brief Description

- One-minute assessment where student reports difference between two numbers for columnar subtraction problems, with minuends varying from 0 to 10.
- A population of the sixty-six unique differences found in Table 6 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the differences onto each form to ensure the content specifications and exceptions below were met for the measure.

**Table 6: Unique Differences for Subtraction from 10 with Minuends Varying from 0 to 10 (66 Unique Differences)**

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 – 0 | – | – | – | – | – | – | – | – | – | – |
| 1 | 1 – 0 | 1 – 1 | – | – | – | – | – | – | – | – | – |
| 2 | 2 – 0 | 2 – 1 | 2 – 2 | – | – | – | – | – | – | – | – |
| 3 | 3 – 0 | 3 – 1 | 3 – 2 | 3 – 3 | – | – | – | – | – | – | – |
| 4 | 4 – 0 | 4 – 1 | 4 – 2 | 4 – 3 | 4 – 4 | – | – | – | – | – | – |
| 5 | 5 – 0 | 5 – 1 | 5 – 2 | 5 – 3 | 5 – 4 | 5 – 5 | – | – | – | – | – |
| 6 | 6 – 0 | 6 – 1 | 6 – 2 | 6 – 3 | 6 – 4 | 6 – 5 | 6 –6 | – | – | – | – |
| 7 | 7 – 0 | 7 – 1 | 7 – 2 | 7 – 3 | 7 – 4 | 7 – 5 | 7 – 6 | 7 – 7 | – | – | – |
| 8 | 8 – 0 | 8 – 1 | 8 – 2 | 8 – 3 | 8 – 4 | 8 – 5 | 8 – 6 | 8 – 7 | 8 – 8 | – | – |
| 9 | 9 – 0 | 9 – 1 | 9 –2 | 9 – 3 | 9 – 4 | 9 – 5 | 9 – 6 | 9 – 7 | 9 – 8 | 9 – 9 | – |
| 10 | 10 – 0 | 10 – 1 | 10 – 2 | 10 – 3 | 10 – 4 | 10 – 5 | 10 – 6 | 10 – 7 | 10 – 8 | 10 – 9 | 10 – 10 |

#### Specifications for Form Content

- Random selection and sequencing, with replacement, of minuends (the "top number") 0–10 and subtrahends (the "bottom number") that produce differences 0–10.
- 80 items per form.

#### Exceptions

1. All possible combinations of 66 differences, with 14 repetitions are used.
2. The same item appearing in the same form more than twice is avoided.
3. Sequences of more than 2 items where one minuend or subtrahend is repeated are avoided (e.g., 5 – 4, 5 – 2 or 4 – 2, 3 – 2 are OK, but not 5 – 4, 5 – 3, 5 – 1).
4. Formula repetition on the same line is avoided.

### Mixed Addition and Subtraction

#### Brief Description

- One-minute assessment where student reports results of random sequence of addition and subtraction problems, with addition problem sums to 20 and subtraction problem minuends to 20.
- A population of the 231 unique sums and 231 unique differences found in Table 3 and Table 3 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the sums and differences onto each form to ensure the content specifications and exceptions below were met for the measure.

**Table 7: Unique Sums and Differences for Mixed Addition and Subtraction (231 Unique Sums)**

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 + 0 | 0 + 1 | 0 + 2 | 0 + 3 | 0 + 4 | 0 + 5 | 0 + 6 | 0 + 7 | 0 + 8 | 0 + 9 | 0 + 10 | 0 + 11 | 0 + 12 | 0 + 13 | 0 + 14 | 0 + 15 | 0 + 16 | 0 + 17 | 0 + 18 | 0 + 19 | 0 + 20 |
| 1 | 1 + 0 | 1 + 1 | 1 + 2 | 1 + 3 | 1 + 4 | 1 + 5 | 1 + 6 | 1 + 7 | 1 + 8 | 1 + 9 | 1 + 10 | 1 + 11 | 1 + 12 | 1 + 13 | 1 + 14 | 1 + 15 | 1 + 16 | 1 + 17 | 1 + 18 | 1 + 19 | – |
| 2 | 2 + 0 | 2 + 1 | 2 + 2 | 2 + 3 | 2 + 4 | 2 + 5 | 2 + 6 | 2 + 7 | 2 + 8 | 2 + 9 | 2 + 10 | 2 + 11 | 2 + 12 | 2 + 13 | 2 + 14 | 2 + 15 | 2 + 16 | 2 + 17 | 2 + 18 | – | – |
| 3 | 3 + 0 | 3 + 1 | 3 + 2 | 3 + 3 | 3 + 4 | 3 + 5 | 3 + 6 | 3 + 7 | 3 + 8 | 3 + 9 | 3 + 10 | 3 + 11 | 3 + 12 | 3 + 13 | 3 + 14 | 3 + 15 | 3 + 16 | 3 + 17 | – | – | – |
| 4 | 4 + 0 | 4 + 1 | 4 + 2 | 4 + 3 | 4 + 4 | 4 + 5 | 4 + 6 | 4 + 7 | 4 + 8 | 4 + 9 | 4 + 10 | 4 + 11 | 4 + 12 | 4 + 13 | 4 + 14 | 4 + 15 | 4 + 16 | – | – | – | – |
| 5 | 5 + 0 | 5 + 1 | 5 + 2 | 5 + 3 | 5 + 4 | 5 + 5 | 5 + 6 | 5 + 7 | 5 + 8 | 5 + 9 | 5 + 10 | 5 + 11 | 5 + 12 | 5 + 13 | 5 + 14 | 5 + 15 | – | – | – | – | – |
| 6 | 6 + 0 | 6 + 1 | 6 + 2 | 6 + 3 | 6 + 4 | 6 + 5 | 6 + 6 | 6 + 7 | 6 + 8 | 6 + 9 | 6 + 10 | 6 + 11 | 6 + 12 | 6 + 13 | 6 + 14 | – | – | – | – | – | – |
| 7 | 7 + 0 | 7 + 1 | 7 + 2 | 7 + 3 | 7 + 4 | 7 + 5 | 7 + 6 | 7 + 7 | 7 + 8 | 7 + 9 | 7 + 10 | 7 + 11 | 7 + 12 | 7 + 13 | – | – | – | – | – | – | – |
| 8 | 8 + 0 | 8 + 1 | 8 + 2 | 8 + 3 | 8 + 4 | 8 + 5 | 8 + 6 | 8 + 7 | 8 + 8 | 8 + 9 | 8 + 10 | 8 + 11 | 8 + 12 | – | – | – | – | – | – | – | – |
| 9 | 9 + 0 | 9 + 1 | 9 + 2 | 9 + 3 | 9 + 4 | 9 + 5 | 9 + 6 | 9 + 7 | 9 + 8 | 9 + 9 | 9 + 10 | 9 + 11 | – | – | – | – | – | – | – | – | – |
| 10 | 10 + 0 | 10 + 1 | 10 + 2 | 10 + 3 | 10 + 4 | 10 + 5 | 10 + 6 | 10 + 7 | 10 + 8 | 10 + 9 | 10 + 10 | – | – | – | – | – | – | – | – | – | – |
| 11 | 11 + 0 | 11 + 1 | 11 + 2 | 11 + 3 | 11 + 4 | 11 + 5 | 11 + 6 | 11 + 7 | 11 + 8 | 11 + 9 | – | – | – | – | – | – | – | – | – | – | – |
| 12 | 12 + 0 | 12 + 1 | 12 + 2 | 12 + 3 | 12 + 4 | 12 + 5 | 12 + 6 | 12 + 7 | 12 + 8 | – | – | – | – | – | – | – | – | – | – | – | – |
| 13 | 13 + 0 | 13 + 1 | 13 + 2 | 13 + 3 | 13 + 4 | 13 + 5 | 13 + 6 | 13 + 7 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 14 | 14 + 0 | 14 + 1 | 14 + 2 | 14 + 3 | 14 + 4 | 14 + 5 | 14 + 6 | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 15 | 15 + 0 | 15 + 1 | 15 + 2 | 15 + 3 | 15 + 4 | 15 + 5 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 16 | 16 + 0 | 16 + 1 | 16 + 2 | 16 + 3 | 16 + 4 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 17 | 17 + 0 | 17 + 1 | 17 + 2 | 17 + 3 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 18 | 18 + 0 | 18 + 1 | 18 + 2 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 19 | 19 + 0 | 19 + 1 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 20 | 20 + 0 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |

**Table 8: Unique Differences for Mixed Addition and Subtraction (231 Unique Differences)**

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 – 0 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 1 | 1 – 0 | 1 – 1 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 2 | 2 – 0 | 2 – 1 | 2 – 2 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 3 | 3 – 0 | 3 – 1 | 3 – 2 | 3 – 3 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 4 | 4 – 0 | 4 – 1 | 4 – 2 | 4 – 3 | 4 – 4 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 5 | 5 – 0 | 5 – 1 | 5 – 2 | 5 – 3 | 5 – 4 | 5 – 5 | – | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 6 | 6 – 0 | 6 – 1 | 6 – 2 | 6 – 3 | 6 – 4 | 6 – 5 | 6 – 6 | – | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 7 | 7 – 0 | 7 – 1 | 7 – 2 | 7 – 3 | 7 – 4 | 7 – 5 | 7 – 6 | 7 – 7 | – | – | – | – | – | – | – | – | – | – | – | – | – |
| 8 | 8 – 0 | 8 – 1 | 8 – 2 | 8 – 3 | 8 – 4 | 8 – 5 | 8 – 6 | 8 – 7 | 8 – 8 | – | – | – | – | – | – | – | – | – | – | – | – |
| 9 | 9 – 0 | 9 – 1 | 9 – 2 | 9 – 3 | 9 – 4 | 9 – 5 | 9 – 6 | 9 – 7 | 9 – 8 | 9 – 9 | – | – | – | – | – | – | – | – | – | – | – |
| 10 | 10 – 0 | 10 – 1 | 10 – 2 | 10 – 3 | 10 – 4 | 10 – 5 | 10 – 6 | 10 – 7 | 10 – 8 | 10 – 9 | 10 – 10 | – | – | – | – | – | – | – | – | – | – |
| 11 | 11 – 0 | 11 – 1 | 11 – 2 | 11 – 3 | 11 – 4 | 11 – 5 | 11 – 6 | 11 – 7 | 11 – 8 | 11 – 9 | 11 – 10 | 11 – 11 | – | – | – | – | – | – | – | – | – |
| 12 | 12 – 0 | 12 – 1 | 12 – 2 | 12 – 3 | 12 – 4 | 12 – 5 | 12 – 6 | 12 – 7 | 12 – 8 | 12 – 9 | 12 – 10 | 12 – 11 | 12 – 12 | – | – | – | – | – | – | – | – |
| 13 | 13 – 0 | 13 – 1 | 13 – 2 | 13 – 3 | 13 – 4 | 13 – 5 | 13 – 6 | 13 – 7 | 13 – 8 | 13 – 9 | 13 – 10 | 13 – 11 | 13 – 12 | 13 – 13 | – | – | – | – | – | – | – |
| 14 | 14 – 0 | 14 – 1 | 14 – 2 | 14 – 3 | 14 – 4 | 14 – 5 | 14 – 6 | 14 – 7 | 14 – 8 | 14 – 9 | 14 – 10 | 14 – 11 | 14 – 12 | 14 – 13 | 14 – 14 | – | – | – | – | – | – |
| 15 | 15 – 0 | 15 – 1 | 15 – 2 | 15 – 3 | 15 – 4 | 15 – 5 | 15 – 6 | 15 – 7 | 15 – 8 | 15 – 9 | 15 – 10 | 15 – 11 | 15 – 12 | 15 – 13 | 15 – 14 | 15 – 15 | – | – | – | – | – |
| 16 | 16 – 0 | 16 – 1 | 16 – 2 | 16 – 3 | 16 – 4 | 16 – 5 | 16 – 6 | 16 – 7 | 16 – 8 | 16 – 9 | 16 – 10 | 16 – 11 | 16 – 12 | 16 – 13 | 16 – 14 | 16 – 15 | 16 – 16 | – | – | – | – |
| 17 | 17 – 0 | 17 – 1 | 17 – 2 | 17 – 3 | 17 – 4 | 17 – 5 | 17 – 6 | 17 – 7 | 17 – 8 | 17 – 9 | 17 – 10 | 17 – 11 | 17 – 12 | 17 – 13 | 17 – 14 | 17 – 15 | 17 – 16 | 17 – 17 | – | – | – |
| 18 | 18 – 0 | 18 – 1 | 18 – 2 | 18 – 3 | 18 – 4 | 18 – 5 | 18 – 6 | 18 – 7 | 18 – 8 | 18 – 9 | 18 –10 | 18 – 11 | 18 – 12 | 18 – 13 | 18 – 14 | 18 – 15 | 18 – 16 | 18 – 17 | 18 – 18 | – | – |
| 19 | 19 – 0 | 19 – 1 | 19 – 2 | 19 – 3 | 19 – 4 | 19 – 5 | 19 – 6 | 19 – 7 | 19 – 8 | 19 – 9 | 19 – 10 | 19 – 11 | 19 – 12 | 19 – 13 | 19 – 14 | 19 – 15 | 19 – 16 | 19 – 17 | 19 – 18 | 19 – 19 | – |
| 20 | 20 – 0 | 20 – 1 | 20 – 2 | 20 – 3 | 20 – 4 | 20 – 5 | 20 – 6 | 20 – 7 | 20 – 8 | 20 – 9 | 20 – 10 | 20 – 11 | 20 – 12 | 20 – 13 | 20 – 14 | 20 – 15 | 20 – 16 | 20 – 17 | 20 – 18 | 20 – 19 | 20 – 20 |

### Specifications for Form Content

- Random sequence of
    a. addition problems with sums to 20 (e.g., random selection, with replacement, of addends 0–20 in columnar addition, with sums up to 20), and
    b. subtraction problems with minuends no greater than 20 and differences 0–20 (e.g., random selection, with replacement, of minuends 0–20 and subtrahends that produce differences 0–20).
- 80 items per form.
- Half of items will be addition and half will be subtraction.

### Exceptions

1. For addition problems, associative replications adjacent in sequence are avoided.
2. For addition problems, sequences of more than two items where one addend is repeated are avoided.
    a. No repeated top addend: 1 + 19, 1 + 5, 1 + 10.
    b. No repeated bottom addend: 2 + 18, 0 + 18, 1 + 18.
3. For subtraction problems, sequences of more than two items where one minuend or subtrahend is repeated (e.g., 5 – 4, 5 – 2 or 4 – 2, 3 – 2 are OK, but 5 – 4, 5 – 3, 5 – 1 is not) are avoided.
4. For all problems, sequences of 3 items of only addition or subtraction are avoided.
5. The same item appearing on the same form is avoided (e.g., 5 – 1 = and 5 – 1 = on the same form; there should be 80 unique items per form).

## Multiplication to 100

### Brief Description

- One-minute assessment where student reports results of multiplication problems, with factors 0 to 10 and products no greater than 100.
- A population of the 121 unique products found in Table 9 served as the source for all 20 forms in the measure; a randomizer program, with manual adjustment where needed, was used to determine the placement of the products onto each form to ensure the content specifications and exceptions below were met for the measure.

**Table 9: Multiplication to 100 (121 Unique Products)**

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 × 0 | 0 × 1 | 0 × 2 | 0 × 3 | 0 × 4 | 0 × 5 | 0 × 6 | 0 × 7 | 0 × 8 | 0 × 9 | 0 × 10 |
| 1 | 1 × 0 | 1 × 1 | 1 × 2 | 1 × 3 | 1 × 4 | 1 × 5 | 1 × 6 | 1 × 7 | 1 × 8 | 1 × 9 | 1 × 10 |
| 2 | 2 × 0 | 2 × 1 | 2 × 2 | 2 × 3 | 2 × 4 | 2 × 5 | 2 × 6 | 2 × 7 | 2 × 8 | 2 × 9 | 2 × 10 |
| 3 | 3 × 0 | 3 × 1 | 3 × 2 | 3 × 3 | 3 × 4 | 3 × 5 | 3 × 6 | 3 × 7 | 3 × 8 | 3 × 9 | 3 × 10 |
| 4 | 4 × 0 | 4 × 1 | 4 × 2 | 4 × 3 | 4 × 4 | 4 × 5 | 4 × 6 | 4 × 7 | 4 × 8 | 4 × 9 | 4 × 10 |
| 5 | 5 × 0 | 5 × 1 | 5 × 2 | 5 × 3 | 5 × 4 | 5 × 5 | 5 × 6 | 5 × 7 | 5 × 8 | 5 × 9 | 5 × 10 |
| 6 | 6 × 0 | 6 × 1 | 6 × 2 | 6 × 3 | 6 × 4 | 6 × 5 | 6 × 6 | 6 × 7 | 6 × 8 | 6 × 9 | 6 × 10 |
| 7 | 7 × 0 | 7 × 1 | 7 × 2 | 7 × 3 | 7 × 4 | 7 × 5 | 7 × 6 | 7 × 7 | 7 × 8 | 7 × 9 | 7 × 10 |
| 8 | 8 × 0 | 8 × 1 | 8 × 2 | 8 × 3 | 8 × 4 | 8 × 5 | 8 × 6 | 8 × 7 | 8 × 8 | 8 × 9 | 8 × 10 |
| 9 | 9 × 0 | 9 × 1 | 9 × 2 | 9 × 3 | 9 × 4 | 9 × 5 | 9 × 6 | 9 × 7 | 9 × 8 | 9 × 9 | 9 × 10 |
| 10 | 10 × 0 | 10 × 1 | 10 × 2 | 10 × 3 | 10 × 4 | 10 × 5 | 10 × 6 | 10 × 7 | 10 × 8 | 10 × 9 | 10 × 10 |

### Specifications for Form Content

- Random selection and sequencing, with replacement, of factors 0–10 in columnar multiplication.
- The digits 0 to 10 will occur randomly as top or bottom factor in columnar multiplication problems with products of 100 or less.
- “Associative” replications (e.g., 6 × 2 = and 2 × 6 =) may occur in the same form.
- 80 items per form.

### Exceptions

1. Associative replications adjacent in sequence are avoided.
2. Sequences of more than 2 items where one factor is repeated are avoided.
    a. No repeated top factor: 1 × 2, 1 × 5, 1 × 7.
    b. No repeated bottom factor: 2 × 8, 0 × 8, 1 × 8.
3. The same item appearing on the same form is avoided (e.g. 3 × 2 = and 3 × 2= on the same form).
4. There should be 80 unique items per form.

# Equating and Scale Development

## Background

In early 2019, the development of Star CBM Math began with the creation of forms for seven different measures: Numeral Recognition, Quantity Comparison, Addition to 10, Addition to 20, Subtraction from 10, Mixed Addition and Subtraction, and Multiplication to 100. Twenty forms were written for each measure. More specific details on each of the measures can be found in "Star CBM Math Content Specification" on page 11.

As with any test development effort, a field test is a crucial step in the process. Field testing of Star CBM Math began in the fall of 2019. The field test was designed to evaluate psychometric quality of the measures, provide data to equate the measures if needed, and to develop scales for the purposes of reporting scores to students. The field test recruited students, schools, and districts from across the country and consisted of two different studies. The first study was designed to evaluate the use of Star CBM Math as a seasonal universal screening tool, while the second study was designed to evaluate the use of Star CBM Math for the purposes of progress monitoring.

Table 10 provides details on the field test studies that took place during the 2019–2020 school year. The specific measures (and number of measures completed) varied across grades and seasons for the universal screening study. For each measure administered by grade and season, students were scheduled to take two forms. For the progress monitoring study, students only took one form of each measure.

**Table 10: Schedule of Assessments for CBM Field Test**

| Math | Study 1—Universal Screening 2 Forms Per Measure Per Season[a] | | | | | | | | | | | | Study 2—Progress Monitoring 1 Form Per Measure | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Kindergarten | | | First | | | Second | | | Third | | | | | | |
| | F | W | S | F | W | S | F | W | S | F | W | S | K | 1 | 2 | 3 |
| Numeral Recognition (NR) | 2 | 2 | 2 | 2 | | | | | | | | | 1 | 1 | | |
| Quantity Comparison (QC) | 2 | 2 | 2 | 2 | 2 | | | | | | | | 1 | 1 | | |
| Addition to 10 (A10) | | | 2 | 2 | 2 | 2 | 2 | | | | | | 1 | 1 | 1 | |
| Addition to 20 (A20) | | | | | 2 | 2 | 2 | 2 | 2 | | | | | 1 | 1 | |
| Subtraction from 10 (S10) | | | | | | | 2 | 2 | 2 | 2 | | | | | 1 | 1 |
| Mixed Addition and Subtraction (MAS) | | | | | | | | | 2 | 2 | 2 | 2 | | | 1 | 1 |
| Multiplication to 100 (MUL) | | | | | | | | | | 2 | 2 | 2 | | | | 1 |
| **Number of Measures/ Season** | **2** | **2** | **3** | **3** | **3** | **2** | **3** | **2** | **3** | **3** | **2** | **2** | **3** | **4** | **4** | **3** |
| **Forms per Administration** | **4** | **4** | **6** | **6** | **6** | **4** | **6** | **4** | **6** | **6** | **4** | **4** | **3** | **4** | **4** | **3** |

a. The Winter Season for Study 1 involved students taking the same forms on two occasions to help evaluate the test-retest reliability of the measures.

This chapter presents the technical details on the process used to equate each of the different measures and create the equated Correct per Minute (CPM) scores that are reported in the software.

## What is Equating and How Can One Determine If Equating Is Needed?

We know that any math measure is likely to differ, at least in small amounts, from one form or assessment occasion to another. Even though content from each form is selected and sequenced randomly from the same general group of possible items, we know that the same student may encounter a particular set or sequence of items that is easier, or perhaps more difficult, than another set they may have been asked to answer. These differences will lead to small variations in a child's score even though the child's skill is the same on both assessment occasions. To reduce the confusion that may come from score differences due to form difficulty rather than student performance, we *equate* scores from each form.

All Star CBM Math scores are expressed as Correct per Minute (CPM), the number of numerals named or problems solved correctly by the student during a one-minute period (or, in rare instances, an estimate of that score when the child completes a task in less than one minute). The simple count of Correct per Minute is then equated for reporting and decision-making.

Equating is a statistical process to adjust for form-to-form difficulty differences when more than one form of the assessment is used with students (see Kolen & Brennan, 2004). Equating is designed to ensure fairness for all students and help mitigate any potential advantage or disadvantage a student may have based on the specific CBM form they were administered. When forms have been equated, the same equated score represents the same level of knowledge and achievement regardless of the form that the student was administered. For example, an equated Correct per Minute (CPM) score of 40 on the Numeral Recognition measure would represent the same level of performance on each of the twenty different forms of Numeral Recognition.

Important steps in equating test forms are selecting a design to carry out the equating and evaluating whether equating is necessary or not. There are three different designs that one can use to perform equating: the single group design, the randomly equivalent group design, and nonequivalent group design (see Kolen & Brennan, 2004). The randomly equivalent group design is the most appropriate design for Star CBM Math because students did not take all of the measures, and the measures were not designed to have common items; it was the design selected to perform the equating. The randomly equivalent group design requires that students are randomly assigned to different forms of the test and that the groups of students taking each form can be assumed to be equivalent in ability. For the CBM field test, random assignment was used to assign students to the different forms they took in each study and generate groups that could be assumed to be equivalent in ability.

To evaluate whether equating is necessary or not, there are two different strategies that one can apply to the collected field test data to determine whether the forms are sufficiently close to each other without equating. Both methods involve looking at the non-equated CPM scores for different forms of each measure. First, one can look at the difference between the form with the highest mean and the form with the lowest mean and evaluate whether the difference between these two means is statistically significant by determining whether the difference is larger than two times the mean standard error. The mean standard error was computed by first computing the standard error of the mean for each form. This is computed by dividing the standard error by the square root of the sample size n. The mean standard error is then the average of the standard errors of the means across all forms of a measure. Second, one can look at the form means for pairs of forms that were shown together in the universal screening field test study and determine whether any of these differences in form means were statistically significant. Ideally, there would be no significant differences when looking at the minimum and maximum form means or the form means for the form pairs.

Table 11 provides a summary of the evaluations of the need for equating for the different measures using the two different methods for the non-equated CPM scores. Shown in the tables are the total sample sizes across forms, average sample size per form, minimum form mean, maximum form mean, the difference between the maximum and minimum form means, and columns corresponding to the two different evaluation methods. The data shown in Table 11 are from the first administrations for each student from the fall, winter, and spring seasons from the universal screening and progress monitoring studies. Data are collapsed across grades in which a set of forms for that measure were used. Most of the data shown in Table 11 was from the fall and winter seasons since a smaller number of assessments were completed in the spring due to school closures associated with the COVID-19 pandemic.

One can see in Table 11 that data indicate that all measures are good candidates for equating to reduce the differences in form means.

**Table 11: Summary of Evaluations of Need for Equating for Star CBM Math**

| Measure | Total Sample Size Across Forms | Average Sample Size Per Form | Minimum Form Mean | Maximum Form Mean | Difference Between the Maximum and Minimum Form Mean | Form Differences Less Than 2 Times the Mean Standard Error | Zero Form Pairs with Statistically Significant Differences |
|---|---|---|---|---|---|---|---|
| Numeral Recognition | 4,855 | 243 | 34.58 | 39.15 | 4.57 | No | No |
| Quantity Comparison | 6,540 | 327 | 20.56 | 25.58 | 5.02 | No | No |
| Addition to 10 | 5,302 | 265 | 17.39 | 22.11 | 4.72 | No | No |
| Addition to 20 | 5,201 | 260 | 9.89 | 17.38 | 7.49 | No | No |
| Subtraction from 10 | 4,943 | 247 | 16.08 | 20.55 | 4.47 | No | No |
| Mixed Addition and Subtraction | 4,033 | 202 | 9.75 | 14.45 | 4.70 | No | No |
| Multiplication to 100 | 3,671 | 184 | 8.31 | 17.48 | 9.17 | No | No |

# Equating Method

There are several different strategies that one can use to perform equating once one determines that equating is needed. These strategies include item response theory (IRT) methods, such as the Rasch (1960) model, and observed-score methods that utilize the correct counts that students received on the assessments. The strategy to conduct the equating for Star CBM Math is based on an observed-score method known as the circle-arc equating method (Livingston & Kim, 2009).

The circle-arc method was a good choice to equate Star CBM Math because of its capacity to handle small to moderate sample sizes, and because it naturally truncates scores when performing equating such that the highest score equals the highest achievable score on the assessment and lowest score equals 0. The circle-arc method can also be applied in cases in which not all the possible scores on a form are observed. Not observing every possible score on a form is a regular occurrence for CBMs. The circle-arc equating method and calculations for Star CBM Math were carried out in R (R core team, 2020) using the package equate (Albano, 2016).

The circle-arc equating method as applied to Star CBM Math requires several steps to produce the equated scores that are reported in the software. First, the number of correct responses on a form (i.e., the correct count) is recorded for every student for the first test administration that they took in each season across the grades in which a set of forms for a measure were used. Basing the equating on data from all seasons and grades in which a measure is used implies that the equating relationships will be consistent across grades and seasons. The mean across all forms is then determined and the easiest form is identified (i.e., the form with the highest mean score). The easiest form is then set as the base form and all other forms are equated to it using the circle-arc equating method through the identification of the minimum possible correct count, the maximum possible correct count, and the mean correct count on each form and fitting a circle-arc through these three points. For all measures, the minimum possible correct count is 0 and the maximum possible correct count is the number of items on the form (see the "Star CBM Math Content Specification" on page 11 for a listing of the number of items on each form).

After linking all the forms to the easiest base form using the circle-arc equating method, one then can develop a set of equating look-up tables such that for every possible correct count on every form one can determine a corresponding equated correct count. Since the easiest form was selected as the base form, the equated correct count will always be greater than or equal to a student's non-equated correct count. This means that equating will never decrease a student's score in the software; the score will always stay the same or increase after equating. The final step in the equating process is to convert the equated correct count to an equated Correct per Minute (CPM) score by multiplying the equated correct count by 60 and dividing by the total time the student took on the assessment. This equated CPM score is then rounded to the nearest integer. The CPM score provides the number of correct responses a student would be expected to give in one minute's time and is one of the most widely accepted metrics and scales for reporting scores on CBMs.

The equated CPM score is the primary score reported in the software for all the math measures. Equated CPM scores indicate the same level of performance and can be interpreted regardless of the specific form taken in a measure. CPM scores can range from 0 and to positive infinity, although in practice virtually all CPM scores are 80 or less. It is also important to note that the equating relationships and corresponding CPM scores are unique to each measure. This means that a CPM score of, say, 20 for Numeral Recognition does not represent the same level of knowledge and achievement as a CPM score of 20 for Quantity Comparison. CPM scores for different measures are not designed to be directly comparable to each other.

## Evaluation of the Quality of the Equating

An important part of performing equating is evaluating the quality of the equating that was done. There are a couple of different strategies that one can use to evaluate the quality of the equating. One method for evaluating the quality of the equating relationship is to look at the differences in form means before and after equating. Ideally, the differences in form means after equating should be small and dramatically reduced in comparison to the form means prior to equating. One would expect that the difference in form means will not be exactly 0 after equating because the equated CPM scores are rounded to the nearest integer and because the circle-arc equating method truncates the equated correct counts to be within the range of possible correct counts that one can theoretically observe on a form.

Table 12 shows the CPM scores before and after equating for each of the Star CBM Math measures. As would be expected, differences in form means were greatly reduced after equating, differences in form means were small, and forms were more comparable following equating than they were before equating. These results suggest that the circle-arc equating method was a successful approach to create equated CPM scores for Star CBM Math.

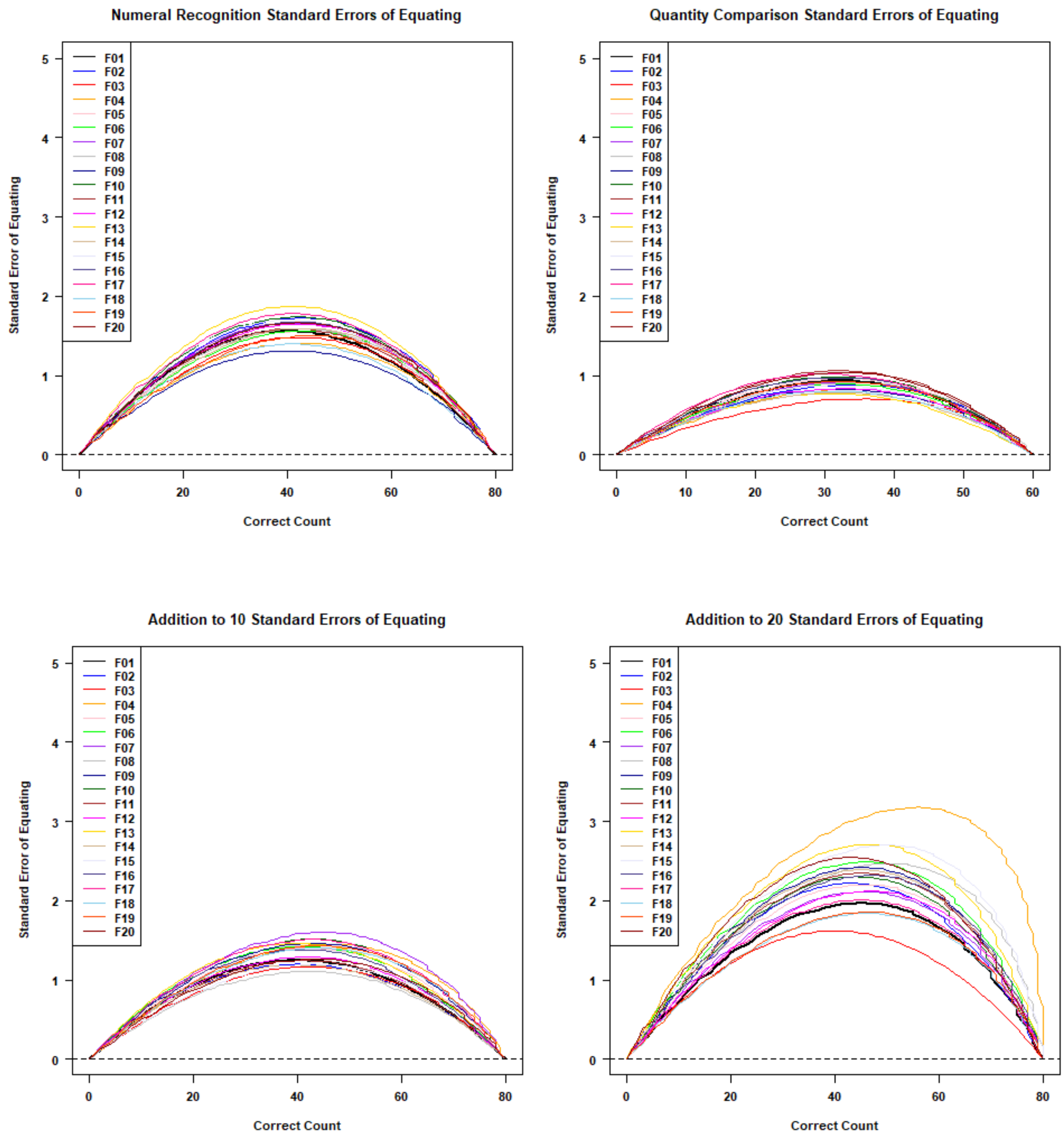
**Table 12: Summary of CPM Form Means Before and After Equating**

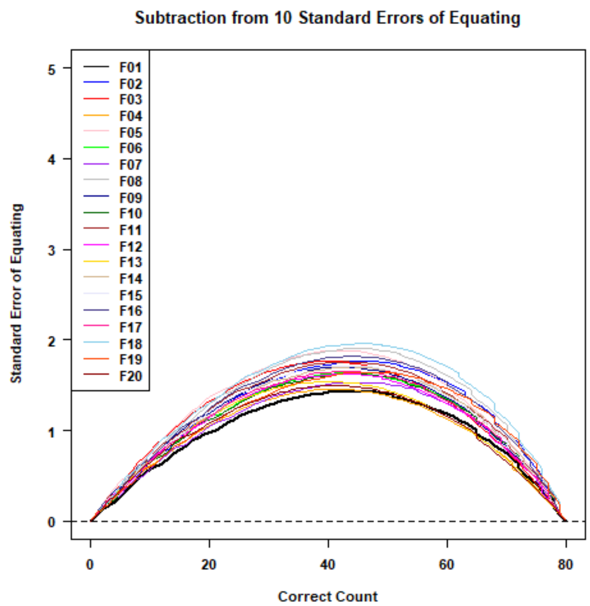
| Measure | Total Sample Size Across Forms | Average Sample Size Per Form | No Equating | | | Equating | | | Reduction in Mean Differences | % Reduction |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Minimum Form Mean | Maximum Form Mean | Difference Between the Maximum and Minimum Form Mean | Minimum Form Mean | Maximum Form Mean | Difference Between the Maximum and Minimum Form Mean | | |
| Numeral Recognition | 4,855 | 243 | 34.58 | 39.15 | 4.57 | 38.32 | 39.47 | 1.15 | 3.42 | 74.8% |
| Quantity Comparison | 6,540 | 327 | 20.56 | 25.58 | 5.02 | 24.97 | 25.70 | 0.73 | 4.29 | 85.5% |
| Addition to 10 | 5,302 | 265 | 17.39 | 22.11 | 4.72 | 21.40 | 22.13 | 0.73 | 3.99 | 84.5% |
| Addition to 20 | 5,201 | 260 | 9.89 | 17.38 | 7.49 | 16.19 | 17.94 | 1.75 | 5.74 | 76.6% |
| Subtraction from 10 | 4,943 | 247 | 16.08 | 20.55 | 4.47 | 20.15 | 20.57 | 0.42 | 4.05 | 90.6% |
| Mixed Addition and Subtraction | 4,033 | 202 | 9.75 | 14.45 | 4.70 | 13.87 | 14.61 | 0.74 | 3.96 | 84.3% |
| Multiplication to 100 | 3,671 | 184 | 8.31 | 17.48 | 9.17 | 16.11 | 17.48 | 1.37 | 7.80 | 85.1% |

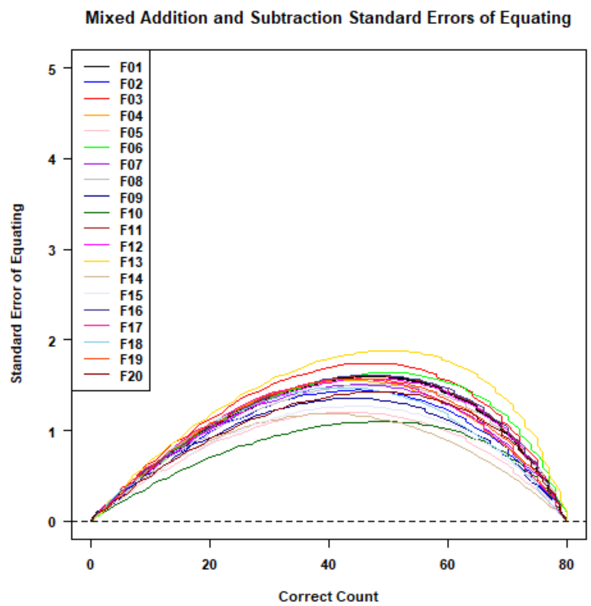
Evaluating change in standard error of equated scores is a second strategy. This standard error of equating describes the amount of random error present in scores after the circle-arc transformation, and is calculated by the analysis package (Albano, 2016). Ideally, the standard error of equated scores should be less than the absolute change from raw to equated scores, indicating that the circle-arc adjustments do not introduce more error into students' scores.

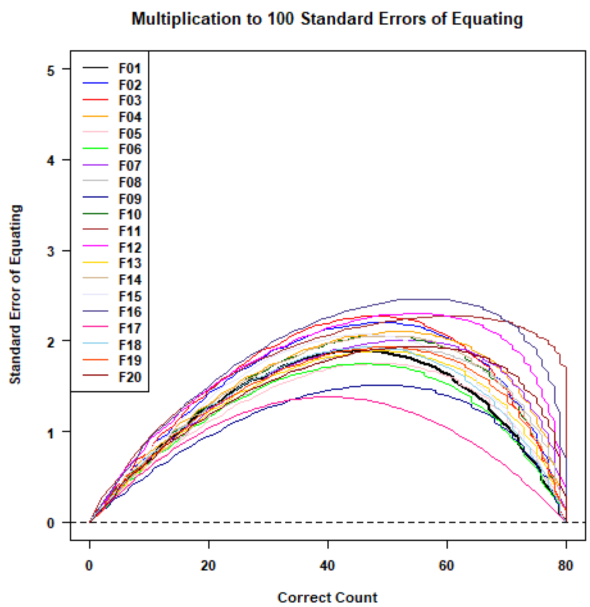
Figure 2 shows the standard errors of equating for the different Star CBM Math measures as a function of correct count. One can see that the standard errors of equating were generally small. For all measures, the standard errors of equating were less than 3.5 and, in many cases, the standard errors of equating were less than 2 across the range of correct counts. It is also important to note that the standard errors of equating shown in Figure 2 were uniformly less than the reduction in mean differences displayed in Table 12. These results also suggest that the circle-arc equating method was a successful approach to create equated CPM scores for the Star CBM Math measures.

**Figure 2: Standard Errors of Equating for Star CBM Math**




Subtraction from 10 Standard Errors of Equating
Standard Error of Equating
Correct Count
F01
F02
F03
F04
F05
F06
F07
F08
F09
F10
F11
F12
F13
F14
F15
F16
F17
F18
F19
F20


Mixed Addition and Subtraction Standard Errors of Equating
Standard Error of Equating
Correct Count
F01
F02
F03
F04
F05
F06
F07
F08
F09
F10
F11
F12
F13
F14
F15
F16
F17
F18
F19
F20


Multiplication to 100 Standard Errors of Equating
Standard Error of Equating
Correct Count
F01
F02
F03
F04
F05
F06
F07
F08
F09
F10
F11
F12
F13
F14
F15
F16
F17
F18
F19
F20

## Other Scoring Considerations

There are some other scoring considerations that warrant discussion as it pertains to equating and using CPM scores. First, it is important to point out that there are other scores besides CPM scores that are reported in the Star CBM Math software. One can find risk categories, attempted per minute scores, percent correct scores, and percentile ranks on screens within the software, score reports, or in data extractions from the software. Each of these scores is described in detail in "Score Definitions" on page 61.

The risk categories and percentile ranks were determined through a norming study and are based on the equated CPM scores for all Star CBM Math measures. There are three risk categories for all measures: Intervention, On Watch, and At/Above Benchmark. The percentile ranks range from 1 to 99, although for many measures not every percentile rank from 1 to 99 is observed because most measures have total scores less than 100. More information on the norming process used to develop the risk categories and percentile ranks can be found in "Norming" on page 51.

While not reported directly in Star CBM student or classroom reports, more detailed data extracts of test results will include attempted per minute scores and percentage correct scores. These scores are not based on equated CPM scores. The attempted per minute scores are determined by applying the same equating look-up table found for the equated correct count to the number of items that a student attempted on the assessment (i.e., the attempted count) to determine an equated attempted count score. This score is then multiplied by 60 and divided by the total time the student took on the assessment to calculate the equated attempted per minute score. Investigations of the field test data suggested that these two scores tended to be highly correlated with each other (all correlations were above 0.95), which lends additional support to using the same equating look-up table with both scores. The equated attempted per minute scores can range from 0 and to positive infinity, although most equated attempted per minute scores are 300 or less. The attempted per minute score provides the number of items a student would be expected to be able to answer in one minute's time.

The percentage correct score is simply 100 times the ratio of the equated correct count to the equated attempted count. The percentage correct provides a measure of the student's accuracy when completing the measure and ranges from 0% to 100%.

Another feature of the Star CBM Math software is the ability to set a goal after the student completes a seasonal screening and monitor a student's progress toward that goal. The teacher first selects a measure for the goal setting and progress monitoring; typically, this will be the measure used for seasonal screening, but other (typically easier) measures can be selected. If the screening measure is selected, the goal-setting wizard in Star CBM software will report the student's most recent equated CPM score, and offer three goal options—a moderate goal, an ambitious goal, or a custom goal (with an open box for setting the custom goal). The target equated CPM scores shown when selecting different goals are determined based on the duration of the goal. The weekly growth rates shown when selecting different types of goals are also in the metric of equated CPM, and in the cases of moderate and ambitious goals are conditional on the specific decile the student scored in when they last took the measure. For a custom goal, the weekly growth rate is determined by taking the difference between the selected target equated CPM and the current equated CPM score and how long the teacher wants the goal to last. Deciles break the percentile ranks into ten different groups (PRs 1 to 10, 11 to 20, 21 to 30, 31 to 40, 41 to 50, 51 to 60, 61 to 70, 71 to 80, 81 to 90, and 91 to 99) and are useful for identifying groups of students that had similar levels of performance on the assessments. A moderate goal is one we expect would be achieved by the top 50 percent of students at that decile, while an ambitious goal is one we expect would be achieved by the top 25 percent of students at that decile. More details on the growth rates and how they were estimated can be found in "Norming" on page 51.

# Reliability and Measurement Precision

Measurement is subject to error. A measurement that is subject to a great deal of error is said to be *imprecise*; a measurement that is subject to relatively little error is said to be *reliable*. In psychometrics, the term *reliability* refers to the degree of measurement precision, expressed as a proportion. A test with perfect score precision would have a reliability coefficient equal to 1, meaning that 100 percent of the variation among persons' scores is attributable to variation in the attribute the test measures, and none of the variation is attributable to error. Perfect reliability is probably unattainable in educational measurement; for example, a test with a reliability coefficient of 0.90 is more likely. On such a test, 90 percent of the variation among students' scores is attributable to the attribute being measured, and 10 percent is attributable to errors of measurement. Another way to think of score reliability is as a measure of the consistency of test scores.

In Star CBM Math, reliability as a measure of consistency between two different measurements indicates the extent to which a test yields consistent results from one administration to another (test-retest reliability) and from one test form to another (alternate forms reliability). Tests must yield somewhat consistent results in order to be useful; the reliability coefficient is obtained by calculating the coefficient of correlation between students' scores on two different occasions, or on two alternate versions of the test given at the same occasion.

There are a variety of methods of estimating the reliability coefficient of a test. Coefficients of correlation calculated between scores on alternate forms, or on similar tests administered two or more times on different occasions, are used to assess alternate forms reliability and test-retest reliability (stability). Coefficients based on psychometric models include *G* coefficients from generalizability theory models.

The Star CBM Math tests provide two ways to evaluate the reliability of scores: reliability coefficients, which indicate the overall precision of a set of test scores as a percentage, and standard errors of measurement (SEM), which provide an index of the degree of error in a test score, expressed on the same scale as the score. A reliability coefficient is a summary statistic that reflects the average amount of measurement precision in a specific examinee group or in a population as a whole. While a reliability coefficient is a single value that applies to the test in general, the magnitude of the SEM reflects random error in the test score.

This chapter presents three different types of reliability coefficients: alternate forms, test-retest, and *G* coefficients from generalizability theory analyses. This is followed by statistics on the standard error of measurement of Star CBM Math test scores.

# Star CBM Math Measures

## G Theory Reliability

In classical test theory, test reliability is generally defined as the proportion of test score variance that is attributable to true variation in the trait the test measures. This can be expressed analytically as

$$Reliability = 1 - \frac{\sigma^2_{error}}{\sigma^2_{total}}$$

where $\sigma^2_{error}$ is the variance of the errors of measurement, and $\sigma^2_{total}$ is the variance of test scores.

This is, however, just one approach to estimating the reliability of test scores. An analogous approach is through what is referred to as generalizability theory, or *G* theory for short. This latter approach does not assume that all of the error in a test is random and undifferentiable; rather that error can be parsed into different components (facets) that are characteristic of a measurement occasion. For example, a student's score can vary due to the student themselves, items encountered on a test, the test form administered, or even the occasion when they tested. All of these situations can have an effect on score variation and can be estimated separately.

While the focus of *G* theory is the interpretation of the variance of the various error components and exploring possible ways of minimizing error, a reliability coefficient known as the generalizability coefficient (G coefficient) is also computed.

It is expressed analytically as

$$Reliability\ (G\ Coefficient) = \frac{\sigma^2_{\rho}}{\sigma^2_{\rho} + \sigma^2_{\delta}}$$

where $\sigma^2_{\rho}$ is the universe score variance and $\sigma^2_{\delta}$ is the error variance. The universe score is the person variance akin to the true score in classical test theory.

The *G* coefficient is interpreted in the same manner as the classical test theory reliability coefficient. It is the proportion of the observed score variance that is attributable to the persons universe score variance. In Star CBM Math, the variance components were estimated in a linear mixed effects model using the lme4 package (Bates, Mächler, Bolker, & Walker, 2015) in the R software (R core team, 2020).

The equation was

Equated Correct per Minute (CPM) = (1|Form)+(1|Person)+(1|Season)+(1|Form.Order)+(1|Form:Form.Order)+(1|Testdays)

where the random effects were estimated for each form, person, form order, form by form order interaction, season, and the number of days since the start of the field test (October 1, 2019). The *G* coefficient was then simply the ratio of the persons variance components to the total variance of persons and the residuals.

Using this technique with the Star CBM Math fall 2019 data resulted in the *G* coefficient reliability estimates shown in Table 13 on page 39.

Results in Table 13 indicate that the *G* coefficients were generally in the 0.70s and 0.80s. Addition to 20 and Subtraction from 10, both in grade 2, had slightly lower coefficients in the 0.60s.

As the table shows, Star CBM Math reliability as indicated by the *G* coefficients is acceptably high for the measures grade by grade.

## Alternate Forms Reliability

Another method of evaluating the reliability of a test is to administer two parallel forms of the same measure to the same examinee at the same time. Parallel forms are matched in content and approximate form difficulty. Next, a reliability coefficient is obtained by calculating the correlation between the two sets of test scores. This is called an alternate forms or parallel forms reliability coefficient.

The alternate forms reliability study provided estimates of Star CBM Math reliability by administering parallel forms to students within the same day during the 2019–2020 school year. All the data were collected between October 1, 2019 and March 31, 2020. The correlation coefficient between the equated Correct per Minute (CPM) scores on the two forms was taken as the reliability estimate for each measure.

Table 13 includes the within-grade alternate forms reliability for all of the Star CBM Math measures.

Results indicate that the alternate forms coefficients were quite high, mostly in the 0.80s. These reliability coefficients indicate a high degree of reliability for the Star CBM Math forms in each measure across the grades.

## Test-Retest Reliability

Another method of evaluating the reliability of a test is to administer the same test form twice to the same examinee. The two forms are typically administered on two different days several days to several weeks apart. Next, a reliability coefficient is obtained by calculating the correlation between the two sets of equated Correct per Minute (CPM) scores. This is called a test-retest reliability coefficient.

Temporal changes in individuals' performance and growth or decline over time can affect test-retest reliability coefficients, usually making them appreciably lower than the alternate forms reliability coefficients.

The test-retest reliability study provided estimates of Star CBM Math reliability by administering the same test forms to students with a minimum two-week interval in January, February, and March of 2020. The correlation coefficient between the equated CPM scores on the two tests was taken as the reliability estimate.

Table 13 includes the within-grade test-retest reliability for all of the measures, along with an indication of the average number of days between testing occasions. The average number of days between testing occasions ranged from approximately 19 to 22 days.

Results indicated that the test-retest coefficients were mostly in the 0.70s and 0.80s.

Because errors of measurement due to content sampling and temporal changes in individuals' performance can affect this correlation coefficient, this type of reliability estimate provides a conservative estimate of the reliability of a single Star CBM Math administration. In other words, the actual Star CBM Math reliability is likely higher than the test-retest reliability estimates indicate. This is evident in the much higher alternate forms reliability coefficients.

**Table 13: Reliability Estimates from the Star CBM Math 2019–2020 Data**

| Math Measures | Grade | G-Coefficients | | Alternate Forms | | Test-Retest | | |
|---|---|---|---|---|---|---|---|---|
| | | n | ρ | n | ρ | n | ρ | Average Days Between Testing |
| Numeral Recognition | K | 2,926 | 0.84 | 1,463 | 0.91 | 1,149 | 0.86 | 20.74 |
| Quantity Comparison | K | 2,866 | 0.74 | 1,433 | 0.87 | 1,134 | 0.75 | 19.85 |
| Addition to 10 | 1 | 3,286 | 0.75 | 1,643 | 0.83 | 1,510 | 0.77 | 21.48 |
| Numeral Recognition | 1 | 1,522 | 0.88 | 761 | 0.88 | – | – | – |
| Quantity Comparison | 1 | 1,522 | 0.82 | 761 | 0.83 | 1,515 | 0.73 | 19.87 |
| Addition to 10 | 2 | 1,310 | 0.85 | 655 | 0.83 | – | – | – |
| Addition to 20 | 2 | 2,878 | 0.66 | 1,439 | 0.88 | 1,286 | 0.88 | 19.5 |
| Subtraction from 10 | 2 | 2,876 | 0.68 | 1,438 | 0.93 | 1,283 | 0.91 | 19.43 |
| Mixed Addition and Subtraction | 3 | 3,384 | 0.77 | 1,692 | 0.78 | 1,545 | 0.77 | 20.62 |
| Multiplication | 3 | 3,332 | 0.74 | 1,666 | 0.82 | 1,538 | 0.77 | 21.42 |
| Subtraction from 10 | 3 | 1,470 | 0.84 | 735 | 0.84 | – | – | – |

Star CBM Math was designed to be a general outcome measure of math. The documented evidence of reliability of the CPM scores using the various methods indicate acceptable levels of score consistency for use in educational settings.

## Standard Error of Measurement

When interpreting the results of any test instrument, it is important to remember that the equated Correct per Minute (CPM) scores represent estimates of a student's true ability level in the attribute measured—here, math. Star CBM scores, like results from all other educational assessments, are not exact nor absolute measures of performance; all measures contain some degree of measurement error. Nor is a single score infallible in the information that it provides. The standard error of measurement can be thought of as an estimate of the precision of any given score. The standard error of measurement describes the extent to which scores would be expected to fluctuate because of chance. If measurement errors follow a normal distribution, a SEM of 7 means that if a student were tested repeatedly, his or her scores would fluctuate within 7 points of his or her first CPM score about 68 percent of the time, and within 14 points (twice the SEM) roughly 95 percent of the time. Since reliability can also be regarded as a measure of precision, there is a close inverse relationship between the reliability of a test and the standard error of measurement for the scores it produces.

The Star CBM Math tests differ from the Star computerized adaptive tests (CATs) in that the CBM tests report the same SEM value for every examinee on a given measure. In other words, each administration of Star CBM Math does not yield a unique "conditional" SEM (CSEM) that reflects the amount of information estimated to be in the specific combination of items that a student received in his or her individual test. This is because the Star CBM Math measures are administered in fixed forms, not adaptively.

Table 14 contains the estimates of Star CBM Math standard errors of measurement (SEM).

The standard error of measurement is based on the traditional SEM estimation method. The alternate forms reliability and the variance of the equated Correct per Minute (CPM) scores were used to estimate the SEM:

$$SEM = SQRT(1 - \rho)\ \sigma_x$$

where

- SQRT() is the square root operator
- $\rho$ is the estimated alternate forms reliability
- $\sigma_x$ is the standard deviation of the observed scores (in this case, equated CPM Scores)

Table 14 summarizes the distribution of SEM values for the 2019–2020 data, for all of the measures by grade level. The SEM values range from 3 to 5 for all of the math measures.

**Table 14: Standard Error of Measurement for the 2019–2020 Star CBM Math Data**

| Math Measures | Grade | Standard Error of Measurement | |
|---|---|---|---|
| | | n | SEM |
| Numeral Recognition | K | 2,926 | 4 |
| Quantity Comparison | K | 2,866 | 3 |
| Addition to 10 | 1 | 3,286 | 4 |
| Numeral Recognition | 1 | 1,522 | 5 |
| Quantity Comparison | 1 | 1,522 | 4 |
| Addition to 10 | 2 | 1,310 | 4 |
| Addition to 20 | 2 | 2,878 | 4 |
| Subtraction from 10 | 2 | 2,876 | 3 |
| Mixed Addition and Subtraction | 3 | 3,384 | 4 |
| Multiplication to 100 | 3 | 3,332 | 5 |
| Subtraction from 10 | 3 | 1,470 | 4 |

# Validity

Test validity was long described as the degree to which a test measures what it is intended to measure. A more current description is that a test is valid to the extent that there are evidentiary data to support specific claims as to *what* the test measures, the *interpretation* of its scores, and the uses for which it is recommended or applied. Evidence of test validity is often indirect and incremental, consisting of a variety of findings that in the aggregate are consistent with the theory that the test measures the intended construct(s), or is suitable for its intended uses and interpretations of its scores. Determining the validity of a test involves the use of data and other information both internal and external to the test instrument itself.

## Content Validity

One touchstone is content validity, which is the relevance of the test questions to the attributes or dimensions intended to be measured by the test. The content of each Star CBM Math measure creates the foundation for "content validity" for the Star CBM Math assessments. These content validity issues were discussed in detail in "Star CBM Math Content Specification" and were an integral part of measure design for Star CBM Math.

## Construct Validity

Construct validity, which is the overarching criterion for evaluating a test, investigates the extent to which a test measures the construct(s) that it claims to be assessing. One method of establishing construct validity involves the use of data and other information external to the test instrument itself. When two different tests designed to measure the same or similar construct correlate strongly and positively, those correlations are viewed as evidence of construct validity. For internal evidence, the correlations between the different measures of the Star CBM Math assessment should correlate with each other. For external evidence, scores of the Star CBM Math tests should correlate highly with other accepted procedures and measures that are used to determine mathematical achievement and comprehension.

This section deals with both internal and external evidence of the validity of Star CBM Math as an assessment of mathematical knowledge and skills.

# Internal Evidence: Relationships Among Star CBM Math Scores

If we treat the different CBM Math measures as specific instances of a general outcome measure of math, we can explore the internal structure of Star CBM Math and the fit of individual measures to this general outcome model by examining relations among different measures when administered to the same students.

Evidence is needed to verify that each Star CBM Math test provides both different and useful information for student mathematical achievement. If the subscores are completely orthogonal—that is, no correlations among different measures—this would suggest that some or all of the measures are actually assessing different, unrelated skills. If the subscores are highly correlated, that would suggest that all of the Star CBM Math tests measure the exact same construct and there is no need to have separate tests.

Table 15 presents the observed correlation matrix of the Star CBM Math measures and the number of students (in parenthesis) taking each measure at each grade. It should be noted that not all Star CBM Math measures are administered at every grade; certain measures are only administered at a particular grade, and therefore correlations can only be computed when two measures are sampled in the same grade. We also note that, by design, we administered most measures in seasons and/or grades where we assumed they would be either too difficult for most students or, conversely, too easy for most students. This decision allowed us to locate grades where measures had maximum utility (i.e., avoiding “floor” or “ceiling” effects), but also may have created circumstances where variance in one or more measures (either because it was too easy or too difficult) would drastically reduce correlations with other measures.

In Grade K, the correlation between Numeral Recognition and Quantity Comparison is 0.69. In Grade 1, correlations among Numeral Recognition, Quantity Comparison and Addition to 10 ranged from 0.54 to 0.63. In Grade 2, correlations among Addition to 10, Addition to 20, and Subtraction from 10 ranged from 0.63 to 0.85. In Grade 3, the correlations among Subtraction from 10, Mixed Addition and Subtraction, and Multiplication to 100 ranged from 0.51 to 0.73.

The correlations were all positive and are all above 0.50. Measures that correlated highly, such as Addition to 10 and Addition to 20, are more closely related as one might expect. For instance, if a student can add to 10, they are likely to also readily transition to addition to 20. In the instances where the correlations between the measures are lower, this is an indication that those measures assess more distinct aspects of the general construct. This is desirable as such measures are informative about the distinct skillsets that must develop for a student to comprehend math.

As expected, some of the correlations are at levels that suggested either lack of relation among measures and/or lack of variance in one or both correlates. There are also instances where correlations may be affected by relatively large measurement error at the test level, given the limited number of items from which the scores were derived and small sample sizes. Consequently, over-interpretation of these correlations, as either high or low, should be made cautiously as well.

**Table 15: Correlations Between Star CBM Math Measures**

| Grade | Measure | Numeral Recognition | Quantity Comparison | Addition to 10 | Addition to 20 | Subtraction from 10 | Mixed Addition and Subtraction | Multiplication to 100 |
|---|---|---|---|---|---|---|---|---|
| K | Numeral Recognition | 1.00 (1,250) | – | – | – | – | – | – |
| | Quantity Comparison | 0.69 (1,249) | 1.00 (1,249) | – | – | – | – | – |
| 1 | Numeral Recognition | 1.00 (801) | – | – | – | – | – | – |
| | Quantity Comparison | 0.63 (798) | 1.00 (1,572) | – | – | – | – | – |
| | Addition to 10 | 0.54 (790) | 0.61 (1,561) | 1.00 (1,599) | – | – | – | – |
| 2 | Addition to 10 | – | – | 1.00 (724) | – | – | – | – |
| | Addition to 20 | – | – | 0.7 (722) | 1.00 (1,398) | – | – | – |
| | Subtraction from 10 | – | – | 0.63 (722) | 0.85 (1,395) | 1.00 (1,397) | – | – |
| 3 | Subtraction from 10 | – | – | – | – | 1.00 (867) | – | – |
| | Mixed Addition and Subtraction | – | – | – | – | 0.73 (863) | 1.00 (1,715) | – |
| | Multiplication to 100 | – | – | – | – | 0.52 (852) | 0.51 (1,688) | 1.00 (1,690) |

**Note:** Numbers in parenthesis are the sample sizes.

# External Evidence: Relationship of Star CBM Math Scores to Scores on Other Tests of Mathematical Achievement

Correlation analysis between Star CBM Math and other Star tests (e.g., Star Math) have been conducted. Students who took both the CBM and Star tests within 30 days were selected for the studies to examine the correlations between Star CBM Math and other Star tests.

Table 16 presents correlations between Star CBM Math and Star Math tests taken between September 1, 2019 to June 1, 2020. Correlations ranged from a low of 0.41 to a high of 0.64 with an average of 0.51. On average, the number of days between taking the two tests was 12 days. In general, these correlation coefficients reflect very well on the validity of the Star CBM Math test as a tool for assessing achievement, screening, and progress monitoring. These validity results, combined with the supporting evidence of reliability and minimization of SEM estimates for the Star CBM Math test, provide a quantitative demonstration of how well this instrument in mathematical achievement assessment performs.

**Table 16: Correlations Between Star CBM Math Measures and Star Math**

| | | Star Math | | |
|---|---|---|---|---|
| **CBM Measure** | **Grade** | **N** | **Average Number of Days Between Tests** | **r** |
| Numeral Recognition | K | 0 | – | – |
| Quantity Comparison | | 0 | – | – |
| Numeral Recognition | 1 | 474 | 14.69 | 0.47 |
| Quantity Comparison | | 1,007 | 12.07 | 0.47 |
| Addition to 10 | | 1,000 | 12.08 | 0.56 |
| Addition to 10 | 2 | 487 | 12.58 | 0.62 |
| Addition to 20 | | 1,028 | 11.26 | 0.42 |
| Subtraction from 10 | | 1,026 | 11.23 | 0.41 |
| Subtraction from 10 | 3 | 741 | 12.6 | 0.64 |
| Mixed Addition and Subtraction | | 1,412 | 11.12 | 0.55 |
| Multiplication to 100 | | 1,397 | 11.04 | 0.49 |

# Classification Accuracy of Star CBM Math

## Accuracy for Identifying At-Risk Students

One of the proposed uses of Star CBM Math is to identify students considered "at risk" for math difficulties and thus requiring supplemental intervention. This screening happens well in advance of state accountability assessment at the end of the school year, which helps educators plan education interventions that can improve students' outcomes. In such cases, correlation coefficients are of lesser interest than classification accuracy statistics, such as overall accuracy of classification, sensitivity, and specificity, among others.

Star CBM Math classification analyses were performed using data obtained between September 1, 2019 and June 1, 2020.

For Star CBM Math measures, the cutpoint for risk is based on the 20th percentile from the norming study (see "Norming" on page 51). Students were classified into two categories based on the scores associated with the 20th percentile for a test. The following indices were calculated for the classification analysis:

1. Sensitivity. Sensitivity refers to the rate at which Star CBM Math identifies students as being at-risk who demonstrate a poor learning outcome at a later point in time. Sensitivity can be thought of as the true positive rate. Screening tools with high sensitivity help ensure that students who truly need intervention will be identified to receive it.
2. Specificity. Specificity refers to the rate at which Star CBM Math identifies students as being not at-risk who perform satisfactorily at a later point in time. Specificity can be thought of as a true negative rate. Screening tools with high specificity help ensure that scarce resources are not invested in students who do not require extra assistance.
3. Area under the ROC (Receiver Operating Characteristic) curve is a powerful indicator of overall accuracy. The ROC curve is a plot of the true positive rate (sensitivity) against the false positive rate (1-specificity) for the full range of possible screener (Star CBM Math) cutpoints. The area under ROC Curve (AUC) is an overall indication of the diagnostic accuracy of the curve. For descriptions of ROC curves and related classification accuracy statistics, refer to Pepe, Janes, Longton, Leisenring, & Newcomb (2004) and Zhou, Obuchowski & McClish (2002).
4. The positive and negative predictive values (PPV and NPV respectively) are the proportions of positive and negative results that are true positive and true negative results, respectively.

AUC values closer to 1 indicate the screening measure reliably distinguishes among students with satisfactory and unsatisfactory performance, whereas values at 0.50 indicate the predictor is no better than chance. According to the National Center on Intensive Intervention (NCII), to be considered convincing evidence, the lower bound of the confidence interval around the Area Under the Curve (AUC) estimate, Sensitivity, and Specificity should be equal to or greater than 0.80. See https://intensiveintervention.org/sites/default/files/NCII_AcademicScreening_RatingRubric_2020-06-30.pdf for details.

Classification diagnostics between Star CBM Math and Star Math are presented in Table 17 below. It can be seen from the table that while Area Under the Curve and Sensitivity values exceed the 0.80 criterion for a few measures, Specificity tend to remain on the lower end, and none of the classification accuracy indices meet the criteria for full bubble.

**Table 17: Classification Diagnostics for Predicting Students' Math Proficiency on the Star Math Assessment from Star CBM Math Scores**

| Star Test | CBM Measure | Grade | Total N | AUC | Sensitivity | Specificity | Positive Predictive Value | Negative Predictive Value |
|---|---|---|---|---|---|---|---|---|
| Star Math | Numeral Recognition | 1 | 401 | 0.76 | 0.83 | 0.54 | 0.98 | 0.10 |
| | Quantity Comparison | | 1,007 | 0.77 | 0.83 | 0.70 | 0.99 | 0.12 |
| | Addition to 10 | | 1,000 | 0.88 | 0.85 | 0.79 | 0.99 | 0.16 |
| | Addition to 10 | 2 | 487 | 0.83 | 0.86 | 0.56 | 0.94 | 0.33 |
| | Addition to 20 | | 1,028 | 0.78 | 0.84 | 0.54 | 0.94 | 0.30 |
| | Subtraction from 10 | | 1,026 | 0.76 | 0.86 | 0.49 | 0.93 | 0.30 |
| | Subtraction from 10 | 3 | 741 | 0.87 | 0.88 | 0.64 | 0.95 | 0.42 |
| | Mixed Addition and Subtraction | | 1,412 | 0.81 | 0.86 | 0.52 | 0.93 | 0.34 |
| | Multiplication to 100 | | 1,397 | 0.74 | 0.83 | 0.51 | 0.93 | 0.29 |

# Evidence of Technical Accuracy for Informing Screening and Progress Monitoring Decisions

Many school districts use tiered models such as Response to Intervention (RTI) or Multi-Tiered Systems of Support (MTSS) to guide instructional decision making and improve outcomes for students. These models represent a more proactive, data-driven approach for better serving students:

- Screen all students to understand where each is in the progression of learning in reading, math, or other disciplines;
- Identify at-risk students for intervention at the earliest possible moment;
- Intervene early for students who are struggling or otherwise at-risk of falling behind; and
- Monitor student progress in order to make decisions as to whether they are responding adequately to the instruction/intervention.

Assessment data are central to both screening and progress monitoring, and Star CBM Math can be used for both purposes. This chapter includes technical information about Star CBM Math's ability to accurately screen students according to risk and to help educators make progress monitoring decisions.

## Screening

According to the US Department of Education, universal screening "is a critical first step in identifying students who are at risk for experiencing...difficulties and who might need more time in instruction or different instruction altogether. Screening is conducted to identify or predict students who may be at risk for poor learning outcomes."[1]

Most commonly, screening is conducted with all students at the beginning of the year and then another two to four times throughout the school year. Star CBM Math can be used for this purpose. In this section, the technical evidence supporting its use to inform screening decisions is summarized.

Organizations offering RTI/MTSS expertise and technical support such as the National Center on Intensive Intervention and the RTI Action Network are generally consistent in how measurement tools should be evaluated for their appropriateness as screeners. Key categories include the following:

1. Validity and reliability. Data on Star CBM Math's reliability were presented in "Reliability and Measurement Precision" on page 35. A wide array of validity evidence has been presented in this chapter and will continue to grow in the coming years as the measures are more widely deployed and additional data are gathered.
2. Practicality and efficiency. Screening measures should not require much teacher or student time. Because most students can complete a Star CBM Math test in one minute or less, and because it is individually administered and scored (see "Introduction" on page 1 for details), Star CBM Math is an exceptionally efficient general outcomes measure for mathematical ability.
3. Classification accuracy metrics including sensitivity, specificity, and overall predictive accuracy. These are arguably the most important indicators, addressing the main purpose of screening: When a brief screening tool indicates a student either is or is not at risk of later math difficulties, how often is it accurate, and what types of errors are made? Evidence supporting Star CBM Math's Classification Accuracy can be found in the preceding portion of this chapter, and much like the validity evidence, this too will be expanded as the measures are deployed and additional data are collected.

1. Source: https://www2.ed.gov/about/inits/ed/earlyliteracy/tools.html

## Progress Monitoring

According to the National Center on Intensive Intervention, "progress monitoring is used to assess a student's performance, to quantify his or her rate of improvement or responsiveness to intervention, to adjust the student's instructional program to make it more effective and suited to the student's needs, and to evaluate the effectiveness of the intervention."[2]

In an RTI/MTSS context, progress monitoring involves frequent assessment—usually occurring once every 1–4 weeks—and often involves only those students who are receiving additional instruction after being identified as at-risk via the screening process. Ultimately, educators use progress monitoring data to determine whether a student is responding adequately to the instruction, or whether adjustments need to be made to the instructional intensity or methods. The idea is to get to a decision quickly, with as little testing as possible, so that valuable time is not wasted on ineffective approaches. Educators make these decisions by comparing their performance against a goal set by the educator. Goals should be "reasonable yet ambitious" as recommended by Shapiro (2008).

The RTI Action Network, National Center on Intensive Intervention, and other organizations offering technical assistance to schools implementing RTI/MTSS models are generally consistent in encouraging educators to use assessments for progress monitoring that have certain characteristics.

A summary of those characteristics and relevant information about Star CBM Math is provided below.

1. **Evidence of psychometric quality.**

    a. **Reliability and validity.** Summaries of the available evidence supporting Star CBM Math's reliability and validity are presented in "Reliability and Measurement Precision" on page 35 and throughout this Validity chapter.

2. **Produce a sufficient number of forms.**

    Approximately 20 forms are available for each Star CBM Math measure, enabling them to be useful for progress monitoring over extended time periods. A variety of grade-specific evidence is available to demonstrate the extent to which Star CBM Math can reliably produce consistent scores across repeated administrations of the same or similar tests to the same individual or group. These include:

2. https://intensiveintervention.org/intensive-intervention/progress-monitor

a. Generic reliability, defined as the proportion of test score variance that is attributable to true variation in the trait or construct the test measures.

b. Alternate forms reliability, defined as the correlation between test scores on repeated administrations to the same examinees.

Grade-level results are summarized in "Reliability and Measurement Precision" on page 35. Finally, scores are equated, further ensuring that observed improvements in student performance can safely be assumed to be a reflection of the student's changing abilities, and not an artifact of the measure itself.

3. **Practicality and efficiency.** As mentioned above, each test takes about one minute to complete, making it an efficient progress monitoring solution.

4. **Specify criterion for adequate growth and benchmarks for end-of-year performance levels.** Benchmarks are suggested for each measure, specifically for each grade and season. Goal-setting decisions are handled by local educators, who know their students best and are familiar with the efficacy and intensity of the instructional supports that will be offered. That said, publishers of assessments used for progress monitoring are expected to provide empirically based guidance to educators on setting goals. The Star CBM Math software offers a goal-setting interface. For measures with norms, the goal tool suggests two goal options that inform educator decision-making. The two suggested options are Moderate (reflecting typical weekly rates of growth for the measure and grade, a rate we would expect 50% of students to meet or exceed) and Ambitious (reflecting above typical rates of weekly growth for the measure and grade, a rate that we would expect only 25% of students to meet or exceed).

## Summary of Star CBM Math Validity Evidence

The validity data presented in this technical manual includes evidence of Star CBM Math's construct validity, as well as classification accuracy statistics. Review of content specifications suggest a close alignment between empirically important aspects of early mathematical competence and measures included in Star CBM Math. Further, correlations both among Star CBM Math measures and between these measures and Star Math provide good evidence of construct (or concurrent) validity. Finally, evaluations of classification accuracy indicate broadly comparable outcomes for identifying individual students' levels of risk for math difficulties. Taken together, these results support the claim that Start CBM Math is a measure of mathematical abilities.

# Norming

Two distinct kinds of norms are described in this chapter: test score norms and growth norms. The former refers to norms based on distributions of test scores at specific points in time. The latter refers to norms looking at changes in distributions of test scores over time; such changes are generally attributed to growth in the attribute that is measured by a test. Hence, norms that describe changes in test scores over time may be called "growth norms."

## Background

Star CBM Math norms were developed in 2020 based on the field testing of the measures in the 2019–2020 school year. The 2020 norms are the ones used in the Star CBM Math software and they will be used until updates are available. This chapter describes the development of the 2020 norms.

The development of the current version of Star CBM Math began in spring of 2019 and includes a range of math measures assessing several different math skills. The specific measures covered by Star CBM Math include Numeral Recognition, Quantity Comparison, Addition to 10, Addition to 20, Subtraction from 10, Mixed Addition and Subtraction, and Multiplication to 100. More details on the Star CBM Math measures can be found in "Star CBM Math Content Specification" on page 11.

Equating was conducted on all of the Star CBM Math measures. The equating was performed to ensure that scores obtained from different forms are comparable. Details on equating can be found in "Equating and Scale Development" on page 24. All the norms were based on equated Correct per Minute (CPM) scores.

Table 18 provides a list of the measures, with the grades and seasons for which norms for those measures are available in the Star CBM Math software.

**Table 18: Summary of Star CBM Math Measures with Grades and Seasons Norms Are Available**

| Measure | Grades and Seasons Norms Are Available |
|---|---|
| Numeral Recognition | All Seasons for Grade K and Fall of Grade 1 |
| Quantity Comparison | All Seasons for Grade K and Grade 1 |
| Addition to 10 | All Seasons for Grade 1 and Grade 2 |
| Addition to 20 | All Seasons for Grade 2 |
| Subtraction from 10 | All Seasons for Grade 2 and Grade 3 |
| Mixed Addition and Subtraction | All Seasons for Grade 3 |
| Multiplication to 100 | All Seasons for Grade 3 |

## The 2020 Norms

The 2020 Star CBM Math norms are based on user data collected from a field test effort that took place in the 2019–2020 school year using a sample of schools from across the US. Due to the COVID-19 outbreak and school closures, very few schools took the measures in spring 2020, and the small amount of data collected was not fully representative of students that participated in the field test in the fall and winter seasons. For these reasons, the norms for all Star CBM Math measures were based on data collected from fall and winter with spring norms determined by extrapolating results from the fall and winter seasons.

Students participating in the norming study took assessments between October 1, 2019 and March 31, 2020. Students took the Star CBM Math tests under normal test administration conditions. No specific norming test was developed, and no deviations were made from the usual test administration. Thus, students in the norming sample took Star CBM Math tests as they are administered in everyday use. However, it should be noted that all the data collected as part of the Star CBM Math field test came from one-on-one administrations between a teacher and student in which the student saw the CBM forms on paper. In the operational version of Star CBM Math, students can take tests on paper as well as online.

Table 19 provides a breakdown of the number of Star CBM Math assessments given in the fall, winter, and spring seasons that were used in the norming analyses. The N counts displayed in Table 19 indicate the total number of assessments given for a particular Star CBM Math measure, and they include data from all of the first administrations of each measure to students in the Star CBM Math field test. It should be noted that the spring season is blank for all measures since these data were not used in the norming analyses.

**Table 19: Numbers of Assessments Per Grade in the Norming Sample**

| Grade | Measure | Fall | Winter | Spring |
|---|---|---|---|---|
| Kindergarten | Numeral Recognition | 1,414 | 1,547 | – |
| | Quantity Comparison | 1,395 | 1,532 | – |
| 1 | Addition to 10 | 1,671 | 1,827 | – |
| | Numeral Recognition | 1,687 | – | – |
| | Quantity Comparison | 1,689 | 1,830 | – |
| 2 | Addition to 10 | 1,444 | 101 | – |
| | Addition to 20 | 1,444 | 1,617 | – |
| | Subtraction from 10 | 1,444 | 1,612 | – |
| 3 | Mixed Addition and Subtraction | 1,686 | 1,997 | – |
| | Multiplication to 100 | 1,652 | 1,976 | – |
| | Subtraction from 10 | 1,697 | 115 | – |

An important part of creating norms is describing the demographic characteristics of the students used in the norms. Information on students' gender and race/ethnicity were obtained from the Renaissance Star CBM customer base. The data shown in Table 20 includes those students from which race/ethnicity or gender were entered and could be matched up to their assessment records. No data are shown for the spring season since these data were interpolated.

Table 20 shows that student gender in each season was close to the national estimates of student gender obtained from the National Center for Educational Statistics (NCES) and Market Data Retrieval (MDR) for the 2018–2019 school year. In terms of race/ethnicity, the norming samples appeared to have a slightly higher percentage of students that were white and lower percentages of many of the other ethnicities in comparison to national estimates.

**Table 20: Student Gender and Ethnicity Information by Season for Star CBM Math**

| Demographic Type | Demographic Group | National Estimate | Fall | Winter | Spring |
|---|---|---|---|---|---|
| Gender | Female | 49.5% | 47.6% | 47.7% | – |
| | Male | 50.5% | 52.4% | 52.3% | – |
| Race/Ethnicity | American Indian | 1.0% | 0.3% | 0.1% | – |
| | Asian | 5.6% | 1.5% | 1.3% | – |
| | Black | 15.2% | 15.7% | 19.3% | – |
| | Hispanic | 27.1% | 10.5% | 11.9% | – |
| | White | 47.1% | 70.3% | 65.9% | – |
| | Multiple Race | 4.1% | 1.7% | 1.5% | – |

Table 21 shows national percentages of students by region, school enrollment, school socio-economic status and location, along with the corresponding percentages in the fall, winter, and spring norming samples. MDR estimates of geographic region were based on four board areas identified by the National Educational Association as Northeastern, Midwestern, Southeastern, and Western regions. The specific states in each region are shown below.

### Geographic Region

Using the categories established by the National Center for Education Statistics (NCES), students were grouped into four geographic regions as defined below: Northeast, Southeast, Midwest, and West.

- **Northeast:** Connecticut, Delaware, District of Columbia, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont
- **Southeast:** Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Virginia, West Virginia
- **Midwest:** Iowa, Illinois, Indiana, Kansas, Michigan, Minnesota, Missouri, North Dakota, Nebraska, Ohio, South Dakota, Wisconsin
- **West:** Alaska, Arizona, California, Colorado, Hawaii, Idaho, Montana, New Mexico, Nevada, Oklahoma, Oregon, Texas, Utah, Washington, Wyoming

### School Size

Based on total school enrollment, schools were classified into one of three school size groups: small schools had under 200 students enrolled, medium schools had between 200–499 students enrolled, and large schools had 500 or more students enrolled.

### Socioeconomic Status

Schools were classified into one of four classifications based on the percentage of students in the school who had free or reduced student lunch. The classifications were coded as follows:

- High socioeconomic status (0%–24%)
- Above-median socioeconomic status (25%–49%)
- Below-median socioeconomic status (50%–74%)
- Low socioeconomic status (75%–100%)

No students were sampled from the schools that did not report the percent of school students with free and reduced lunch.

### Location

Schools were also classified based on the location of the school. There were four different school locations recorded in the MDR data and these four school locations were as follows:

- Rural
- Suburban
- Town
- Urban

**Table 21: Student Demographic Information by Season for Star CBM Math**

| | Region | National Estimates | Fall | Winter | Spring |
|---|---|---|---|---|---|
| Region | Midwest | 26.0% | 41.1% | 48.4% | – |
| | Northeast | 19.4% | 23.2% | 16.1% | – |
| | Southeast | 21.5% | 17.9% | 19.4% | – |
| | West | 33.2% | 17.9% | 16.1% | – |
| School Enrollments | <200 | 24.6% | 26.8% | 29.0% | – |
| | 200–499 | 40.0% | 41.1% | 46.8% | – |
| | ≥500 | 35.5% | 32.1% | 24.2% | – |
| School Socioeconomic Status | Low | 12.2% | 22.5% | 22.6% | – |
| | Below Median | 17.9% | 32.7% | 32.1% | – |
| | Above Median | 19.4% | 26.5% | 26.4% | – |
| | High | 50.6% | 18.4% | 18.9% | – |
| Location | Rural | 23.6% | 48.2% | 48.3% | – |
| | Suburban | 33.1% | 33.9% | 30.0% | – |
| | Town | 14.4% | 12.5% | 16.7% | – |
| | Urban | 28.9% | 5.4% | 5.0% | – |

**Note:** Region and location include all schools (public, non-public, Catholic) in the US; school enrollment and socioeconomic status information is only available for public schools.

While not specifically reported in Table 20 or Table 21, the norming sample also included public and private schools, students with and without disabilities, students who were and were not English Language Learners, and students who were and were not gifted and talented.

## Test Administration

All students took the Star CBM Math tests under normal administration procedures. A normal test administration was one in which a teacher administered the measure one-on-one to a student, with the student seeing the measure on paper. The teacher then recorded the number of items the student answered correct, the number of items they attempted, and the time that they took on the assessment, and these data where then entered into a software application used to record and track the scores obtained on the assessments. Some students in the norming samples took more than one assessment in a season, but the norming analyses used data only from the first test administration that a student took in each season. Depending on the field test study that a student participated in, the first test administration to a student may have included one or two assessments of the different Star CBM Math measures targeted to the student's grade level.

## Data Analysis

The norming analyses used all the data described in Table 19. Separate norms were computed for each grade, season, and measure. No weighting was done based on demographics as part of the norming process. Star CBM Math norms are based on the field test sample described above. While broadly representative of the population of US elementary school students, and thus appropriate for selecting measures and setting benchmarks for levels of risk, these norms will continue to be updated as additional data become available in future months and years.

For all of the measures, the first step in the norming process was to find the equated Correct per Minute (CPM) scores associated with the percentile ranks from 1 to 99 for both the fall and winter data for every grade the measure was used in. The difference between these CPM scores was computed and any score reversals across seasons were eliminated. The mean difference between the winter and fall equated CPM scores across percentile ranks was then calculated and this value was added to winter scores to extrapolate scores for the spring. Extrapolation was needed for the spring data because COVID-19 impacted the spring season for the Star CBM Math field test. A final set of percentile rank tables was then determined by eliminating repeated CPM scores for each measure, grade, and season, keeping the rows that corresponded with the lowest percentile rank observed in the tables for each measure, grade, and season. This final set of percentile rank tables was then used to determine the risk category tables that are used to provide indicators of risk in the Star CBM Math software for each measure, grade, and season. By default, a student is classified as being in the At/Above Benchmark category if the percentile rank associated with their equated CPM score ranges from 40 to 99, in the On Watch category if the percentile rank associated with their equated CPM score ranges from 20 to 39, and in the Intervention category if the percentile rank associated with their equated CPM score ranges from 1 to 19.

For the Star CBM Math measures, a student scoring in the At/Above Benchmark category is shown in green in the software, a student scoring in the On Watch category is shown in blue in the software, and the Intervention category is shown in red.

The percentile rank and risk categories tables determined from the norming analyses can be found in “Conversion Tables” on page 65.

Table 22 provides descriptive statistics for each measure, grade, and season in the equated CPM metric for all measures. No data is shown for the spring season because these data were not used in the norming process. One can see in Table 22 that the mean CPM scores for each measure tend to increase across grades and seasons as expected.

**Table 22: Descriptive Statistics for Star CBM Math Tests Across Seasons**

| Season | Grade | Measure | N | Mean | Std Dev | Median |
|---|---|---|---|---|---|---|
| Fall | Kindergarten | Numeral Recognition | 1,414 | 26 | 14 | 26 |
| | | Quantity Comparison | 1,395 | 16 | 8 | 16 |
| | 1 | Addition to 10 | 1,671 | 18 | 9 | 17 |
| | | Numeral Recognition | 1,687 | 52 | 15 | 52 |
| | | Quantity Comparison | 1,689 | 28 | 9 | 29 |
| | 2 | Addition to 10 | 1,444 | 30 | 11 | 29 |
| | | Addition to 20 | 1,444 | 18 | 9 | 18 |
| | | Subtraction from 10 | 1,444 | 16 | 8 | 15 |
| | 3 | Mixed Addition and Subtraction | 1,686 | 13 | 7 | 12 |
| | | Multiplication to 100 | 1,652 | 14 | 11 | 12 |
| | | Subtraction from 10 | 1,697 | 22 | 10 | 21 |
| Winter | Kindergarten | Numeral Recognition | 1,547 | 35 | 15 | 33 |
| | | Quantity Comparison | 1,532 | 21 | 9 | 21 |
| | 1 | Addition to 10 | 1,827 | 20 | 9 | 18 |
| | | Quantity Comparison | 1,830 | 33 | 9 | 33 |
| | 2 | Addition to 10 | 101 | 34 | 18 | 30 |
| | | Addition to 20 | 1,617 | 23 | 15 | 20 |
| | | Subtraction from 10 | 1,612 | 22 | 14 | 19 |
| | 3 | Mixed Addition and Subtraction | 1,997 | 15 | 8 | 13 |
| | | Multiplication to 100 | 1,976 | 19 | 11 | 18 |
| | | Subtraction from 10 | 115 | 28 | 14 | 25 |

## Identification of Screening Measures

A unique feature of Star CBM Math is the identification of a single measure that can be used for the purposes of initially screening students in each grade and season. For Kindergarten students the screening measure in each season is Numeral Recognition, for grade 1 students the screening measure in each season is Quantity Comparison, for grade 2 students the screening measure in each season is Addition to 20, and for grade 3 students the screening measure in each season is Mixed Addition and Subtraction. The screening measures were identified based on our review of criteria associated with General Outcome Measure design principles, as described in the introductory chapter of this manual. To assure simplicity, we wanted to select measures that were easy to administer and complete with fidelity. To assess rigor, we reviewed descriptive statistics and normative data, reliability estimates, the standard errors of measurement, and validity coefficients with Star Math (CAT) for each measure. The identified screening measures were those that yielded a range of scores without floor or ceiling effects, had high reliability estimates, had low standard errors of measurement, and exhibited high correlations with external tests. Summaries of the descriptive statistics can be found in Table 22, and the normative data can be found in “Conversion Tables” on page 65. Details on the reliability estimates and the standard errors of measurement can be found in “Reliability and Measurement Precision” on page 35, while the estimated validity coefficients can be found in “Validity” on page 41.

It is important to point out that even though a screening measure is identified by the Star CBM Math software, teachers may test their students on other measures if they think those measures are appropriate for their students or if they are looking for additional data insights. The screening measure is identified to provide guidance if a teacher only wants to administer a single test to a student to help get an initial evaluation of their math achievement. Using other available measures can help provide educators with a more complete picture of a student’s math achievement. Table 18 on page 52 displays a summary of Star CBM Math measures that have available norms in different grades and seasons. If an educator tests a student with a measure that has norms, they can obtain color-coded risk categories on different views and reports within the software.

For all measures, teachers can also set a goal for a student for normed measures and they can view weekly growth rates if they are interested in progress monitoring a student with that particular measure. The development of the weekly growth rates (growth norms) is described in the next section. It should be noted that before you can set a goal and progress monitor a student, that student must have already taken a test for that measure.

## Growth Norms

Student achievement is often thought of in terms of status: a student’s performance at one point in time. However, this ignores important information about a student’s learning trajectory—how much students are growing over a period of time. When educators are able to consider growth information—the amount or rate of change over time—alongside current status, a richer picture of the student emerges, empowering educators to make better instructional decisions.

For Star CBM Math, educators can see growth information when they decide to set a goal and progress monitor their students. Goals can only be set for measures in which a student has already taken at least one assessment. Growth information is available for normed Star CBM Math measures.

When a teacher sets a goal, Star CBM will display the CPM score from the student's most recent assessment on that measure and ask the teacher to set an end date for the goal.

The teacher sets this goal using a wizard within the Star CBM software. When viewing this wizard for an individual student, the teacher will see the student's most recent equated CPM score and additional information to help the teacher answer 2 questions. First, the teacher sets when the time period for evaluation of the new goal should end; the goal may extend to the next seasonal screening period, to the end of the current academic year, or to other dates.

Next, the teacher is given three choices for setting the goal itself, which is the CPM score the student should meet by the end of that time period. Choices include moderate, ambitious, and custom goals. For a custom goal, the weekly growth rate shown is determined by first taking the difference between the target CPM score the teacher enters and the current score; that difference is then divided by the number of weeks in the goal period (based on the end date). For moderate and ambitious goals, a weekly growth rate is shown below the target score. Weekly growth rates for moderate and ambitious goals are dependent on the student's decile score for the most recent measure, grade, and season. Deciles break the percentile ranks into ten different groups (PRs 1 to 10, 11 to 20, 21 to 30, etc.) and are useful for identifying groups of students with similar levels of performance. A moderate goal is one we expect would be achieved by the top 50 percent of students at that decile, and an ambitious goal is one we expect would be achieved by the top 25 percent of students at that decile.

To determine the moderate weekly growth rates for all measures, we identified samples of students who took Star CBM Math measures in both fall and winter. The first assessments that these students took in each season were found, and the difference between their winter and fall scores was calculated. This difference score was then divided by the number of days between the two tests, and this value was multiplied by 7 to get a weekly growth rate for each student. Scores were then grouped by decile and all the scores within a decile were sorted. The moderate weekly growth rate for that decile was the growth rate equal to the 50th percentile rank. The process for identifying the ambitious growth rate was the same as for the moderate weekly growth rates except that the ambitious weekly growth rate for that decile was the growth rate equal to the 75th percentile rank. There are separate weekly growth rates by grade, measure, and decile.

# Score Definitions

This chapter enumerates all of the scores reported by Star CBM Math, which include criterion-referenced scores and norm-referenced scores.

## Types of Test Scores

In a broad sense, Star CBM Math provides two types of test scores that measure student performance in different ways:

- *Criterion-referenced scores* describe a student's performance relative to a specific content domain or to a standard. Such scores may be expressed either on a continuous score scale or as a classification. An example of a criterion-referenced score on a continuous scale is a percent-correct score, which expresses what proportion of test questions the student can answer correctly in the content domain. An example of a criterion-referenced classification is a proficiency category on a standards-based assessment: the student may be said to be "proficient" or not, depending on whether the student's score equals, exceeds, or falls below a specific criterion (the "standard") used to define "proficiency" on the standards-based test. The criterion-referenced score reported by Star CBM Math is the Correct per Minute (CPM) score, which is a tally of the number of correct responses on a given task or probe for the duration of one minute.
- *Norm-referenced scores* compare a student's test results to the results of other students who have taken the same test. In this case, scores provide a relative measure of student achievement compared to the performance of a group of students at a given time. Percentile Ranks are the primary norm-referenced scores available in the Star CBM Math software.

### Correct Per Minute (CPM)

Star CBM Math consists of several measures and multiple forms within each measure. In order to make the results of all forms in a measure comparable, and in order to provide a basis for deriving the norm-referenced scores, it is necessary to convert (or equate) all the scores of Star CBM Math forms to a common or base form within each measure. Equating is a statistical process used to eliminate form difficulty differences to allow for interpretation of scores on a common scale. Star CBM Math does this in steps. First, the number of correct responses on a form (Correct Count) is recorded. Second, the easiest form in the same Star CBM Math measure is identified and the

Correct Count score on all of the other forms is equated to the easiest form, referred to as the base form. Finally, the equated correct count is converted to a Correct per Minute (CPM) score for reporting by multiplying the equated correct count by 60 and dividing by the total time the student took on the assessment (in seconds). As a result, CPM scores indicate the same level of performance and can be interpreted regardless of the specific form taken in a measure.

## Attempted Per Minute

The Attempted per Minute is the number of items estimated to be answered in a one-minute duration. A similar process is used to determine Attempted per Minute scores as Correct per Minute scores. First, the attempted count is recorded. Second, the attempted count is equated to the easiest base form using the same transformation as is used for the correct count. Finally, the equated attempted count is converted to an Attempted per Minute score for reporting by multiplying the equated attempted count by 60 and dividing by the total time the student took on the assessment (in seconds).

## Accuracy/Percent Correct

The Accuracy/Percent Correct score is the percentage of items answered correctly out of the total number of items attempted. It is found by dividing the equated correct count divided by the equated attempted count and then multiplying by 100. The Percent Correct provides a measure of how accurate a student was when completing the measure. Percent correct is calculated and required for reporting in some states.

## Percentile Rank (PR)

Percentile Rank is a norm-referenced score that indicates the percentage of students in the same grade and at the same point of time in the school year who obtained scores lower than the score of a particular student. In other words, Percentile Ranks show how an individual student's performance compares to that of his or her same-grade peers in the norming sample. For example, a Percentile Rank of 85 means that the student is performing at a level that exceeds 85 percent of other students in that grade at the same time of the year. Percentile Ranks simply indicate how a student performed compared to the others who took Star CBM Math tests as a part of the norming study. Seasonal (fall, winter, and spring) Percentile Ranks are reported for Star CBM Math. The range of Percentile Ranks is 1–99.

The Percentile Rank scale is not an equal-interval scale. For this reason, PR scores should not be averaged or otherwise algebraically manipulated. NCE scores are much more appropriate for these activities.

Table 30 through Table 36 on page 68 through page 95 contain the CPM to Percentile Rank conversion table that the Star CBM Math software uses for each measure, grade, and season. These tables can be used to estimate PR values for tests that were taken in a particular grade and season for each measure.

## Normal Curve Equivalent (NCE)

Normal Curve Equivalents (NCEs) are scores that have been scaled in such a way that they have a normal distribution, with a mean of 50 and a standard deviation of 21.06 in the normative sample for a given test.

Because they range from 1–99, they appear similar to Percentile Ranks, but they have the advantage of being based on an equal interval scale. That is, the difference between two successive scores on the scale has the same meaning throughout the scale. NCEs are useful for purposes of statistically manipulating norm-referenced test results, such as when interpolating test scores, calculating averages, and computing correlation coefficients between different tests. For example, in Star CBM Math score reports, average Percentile Ranks can be obtained by first converting the PR values to NCE values, averaging the NCE values, and then converting the average NCE back to a PR.

Table 38 on page 99 provides the NCEs corresponding to integer PR values and facilitates the conversion of PRs to NCEs. Table 37 on page 96 provides the conversions from NCE to PR. The NCE values are given as a range of scores that convert to the corresponding PR value.

## Risk Category

The Star CBM Math software assigns students to risk categories based on their Correct Per Minute (CPM) score and the Percentile Rank (PR) associated with their CPM score. By default, the software reports three risk categories for all measures. The risk categories are labeled At/Above Benchmark, On Watch, and Intervention. These three risk categories are signified by the colors of green, blue, and red in the software, and they correspond to the PRs of 40 to 99, 20 to 39, and 1 to 19. The CPM score ranges associated with different risk categories change by measure, grade, and season.

# Seasonal Placement

Star CBM Math software uses the student's grade and predetermined seasons when determining the PRs and risk categories. The fall season is the earlier of the school year start date or August 1 through November; Winter is December 1st through March 31st; and Spring is April 1st through July 31st or the school year end date, whichever is later. Percentile ranks and resulting risk category evaluations are based on Correct per Minute, and vary by season and by grade.

## Compensating for Incorrect Grade

Teachers cannot make retroactive corrections to a student's grade by editing the grade assignments in a student's record after students have tested. In other words, Star CBM Math software cannot go back in time and correct scores resulting from erroneous grade information. Thus, it is extremely important for the test administrator to make sure that students are assigned the correct grade before they take assessments.

# Conversion Tables

## Seasonal Benchmarks for Math: Percentile Ranks and Correct Per Minute Score Ranges Associated with Benchmark Categories by Grade and Season

**Table 23: Numeral Recognition Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | | |
|---|---|---|---|---|---|
| | | Grade K Fall | Grade K Winter | Grade K Spring | Grade 1 Fall |
| At/Above Benchmark | 40 and above | 23 and above | 30 and above | 39 and above | 48 and above |
| On Watch | 20 to 39 | 14 to 22 | 23 to 29 | 32 to 38 | 38 to 47 |
| Intervention | 1 to 19 | 0 to 13 | 0 to 22 | 0 to 31 | 0 to 37 |

**Table 24: Quantity Comparison Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | | | | |
|---|---|---|---|---|---|---|---|
| | | Grade K | | | Grade 1 | | |
| | | Fall | Winter | Spring | Fall | Winter | Spring |
| At/Above Benchmark | 40 and above | 14 and above | 19 and above | 24 and above | 26 and above | 31 and above | 35 and above |
| On Watch | 20 to 39 | 9 to 13 | 13 to 18 | 18 to 23 | 21 to 25 | 25 to 30 | 29 to 34 |
| Intervention | 1 to 19 | 0 to 8 | 0 to 12 | 0 to 17 | 0 to 20 | 0 to 24 | 0 to 28 |

**Table 25: Addition to 10 Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | | | | |
|---|---|---|---|---|---|---|---|
| | | Grade 1 | | | Grade 2 | | |
| | | Fall | Winter | Spring | Fall | Winter | Spring |
| At/Above Benchmark | 40 and above | 15 and above | 16 and above | 18 and above | 27 and above | 27 and above | 31 and above |
| On Watch | 20 to 39 | 10 to 14 | 12 to 15 | 14 to 17 | 20 to 26 | 21 to 26 | 25 to 30 |
| Intervention | 1 to 19 | 0 to 9 | 0 to 11 | 0 to 13 | 0 to 19 | 0 to 20 | 0 to 24 |

**Table 26: Addition to 20 Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | |
|---|---|---|---|---|
| | | Grade 2 Fall | Grade 2 Winter | Grade 2 Spring |
| At/Above Benchmark | 40 and above | 16 and above | 18 and above | 23 and above |
| On Watch | 20 to 39 | 11 to 15 | 13 to 17 | 18 to 22 |
| Intervention | 1 to 19 | 0 to 10 | 0 to 12 | 0 to 17 |

**Table 27: Subtraction from 10 Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | | | | |
|---|---|---|---|---|---|---|---|
| | | Grade 2 | | | Grade 3 | | |
| | | Fall | Winter | Spring | Fall | Winter | Spring |
| At/Above Benchmark | 40 and above | 13 and above | 17 and above | 22 and above | 19 and above | 22 and above | 28 and above |
| On Watch | 20 to 39 | 9 to 12 | 12 to 16 | 17 to 21 | 13 to 18 | 16 to 21 | 22 to 27 |
| Intervention | 1 to 19 | 0 to 8 | 0 to 11 | 0 to 16 | 0 to 12 | 0 to 15 | 0 to 21 |

**Table 28: Mixed Addition and Subtraction Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | |
|---|---|---|---|---|
| | | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
| At/Above Benchmark | 40 and above | 11 and above | 12 and above | 14 and above |
| On Watch | 20 to 39 | 7 to 10 | 8 to 11 | 10 to 13 |
| Intervention | 1 to 19 | 0 to 6 | 0 to 7 | 0 to 9 |

**Table 29: Multiplication to 100 Percentile Ranks and Correct Per Minute Scores Associated with Benchmark Categories**

| Benchmark Category | PRs Associated with Benchmark | Correct Per Minute Scores Associated with Benchmark | | |
|---|---|---|---|---|
| | | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
| At/Above Benchmark | 40 and above | 10 and above | 15 and above | 21 and above |
| On Watch | 20 to 39 | 5 to 9 | 10 to 14 | 16 to 20 |
| Intervention | 1 to 19 | 0 to 4 | 0 to 9 | 0 to 15 |

# Percentile Rank (PR) to Correct Per Minute (CPM) Conversion Tables

**Note:** If your district uses different benchmark categories from the default categories used in Star CBM (see the previous pages), use the following tables to find the Correct Per Minute scores that correspond to the Percentile Ranks that you use for your categories. If one of those Percentile Ranks does not have a matching Correct Per Minute score (you see "—" instead of a score in the table), find the next highest Percentile Rank with an available Correct Per Minute score, and use that Correct per Minute score as the benchmark.

**Table 30: Numeral Recognition—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K Fall | Grade K Winter | Grade K Spring | G1 Fall |
|---|---|---|---|---|
| 1 | – | 0 | 0 | 0 |
| 2 | 0 | 7 | 16 | 22 |
| 3 | 1 | 10 | 19 | 24 |
| 4 | 2 | 11 | 20 | 26 |
| 5 | 3 | 13 | 22 | – |
| 6 | 4 | 15 | 24 | 27 |
| 7 | 5 | 16 | 25 | 29 |
| 8 | 6 | – | – | 30 |
| 9 | – | 17 | 26 | 31 |
| 10 | 7 | 18 | 27 | 32 |
| 11 | 8 | – | – | – |
| 12 | 9 | 19 | 28 | – |
| 13 | 10 | – | – | 33 |
| 14 | – | 20 | 29 | 34 |
| 15 | 11 | – | – | 35 |
| 16 | 12 | – | – | – |
| 17 | – | 21 | 30 | 36 |
| 18 | 13 | 22 | 31 | 37 |
| 19 | – | – | – | – |
| 20 | 14 | 23 | 32 | 38 |
| 21 | – | – | – | 39 |
| 22 | 15 | 24 | 33 | – |
| 23 | 16 | – | – | 40 |
| 24 | – | – | – | – |
| 25 | 17 | 25 | 34 | – |
| 26 | – | – | – | 41 |

**Table 30: Numeral Recognition—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K Fall | Grade K Winter | Grade K Spring | G1 Fall |
|---|---|---|---|---|
| 27 | 18 | — | — | — |
| 28 | — | — | — | 42 |
| 29 | — | 26 | 35 | — |
| 30 | 19 | — | — | 43 |
| 31 | — | — | — | — |
| 32 | — | 27 | 36 | 44 |
| 33 | 20 | — | — | 45 |
| 34 | — | — | — | — |
| 35 | — | 28 | 37 | — |
| 36 | 21 | — | — | 46 |
| 37 | — | — | — | 47 |
| 38 | — | 29 | 38 | — |
| 39 | 22 | — | — | — |
| 40 | — | — | — | 48 |
| 41 | 23 | 30 | 39 | — |
| 42 | — | — | — | 49 |
| 43 | — | — | — | — |
| 44 | — | 31 | 40 | — |
| 45 | 24 | — | — | 50 |
| 46 | — | — | — | — |
| 47 | — | 32 | 41 | — |
| 48 | 25 | — | — | 51 |
| 49 | — | — | — | — |
| 50 | — | 33 | 42 | 52 |
| 51 | 26 | — | — | — |
| 52 | — | — | — | 53 |

**Table 30: Numeral Recognition—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K Fall | Grade K Winter | Grade K Spring | G1 Fall |
|---|---|---|---|---|
| 53 | – | 34 | 43 | – |
| 54 | – | – | – | – |
| 55 | – | – | – | 54 |
| 56 | 27 | 35 | 44 | – |
| 57 | – | – | – | 55 |
| 58 | 28 | 36 | 45 | – |
| 59 | – | – | – | – |
| 60 | – | 37 | 46 | 56 |
| 61 | – | – | – | – |
| 62 | 29 | 38 | 47 | – |
| 63 | – | – | – | 57 |
| 64 | – | – | – | – |
| 65 | 30 | 39 | 48 | – |
| 66 | – | – | – | 58 |
| 67 | 31 | 40 | 49 | – |
| 68 | – | – | – | 59 |
| 69 | – | – | – | – |
| 70 | 32 | 41 | 50 | 60 |
| 71 | – | – | – | – |
| 72 | – | 42 | 51 | 61 |
| 73 | 33 | – | – | – |
| 74 | – | 43 | 52 | 62 |
| 75 | – | – | – | – |
| 76 | 34 | 44 | 53 | 63 |
| 77 | – | – | – | – |
| 78 | 35 | 45 | 54 | – |

**Table 30: Numeral Recognition—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K Fall | Grade K Winter | Grade K Spring | G1 Fall |
|---|---|---|---|---|
| 79 | 36 | – | – | 64 |
| 80 | – | 46 | 55 | – |
| 81 | 37 | 47 | 56 | 65 |
| 82 | 38 | – | – | – |
| 83 | 39 | 48 | 57 | 66 |
| 84 | – | – | – | – |
| 85 | 40 | 49 | 58 | 67 |
| 86 | 41 | 50 | 59 | 68 |
| 87 | – | – | – | – |
| 88 | 42 | 51 | 60 | 69 |
| 89 | 43 | 52 | 61 | 70 |
| 90 | 45 | 54 | 63 | 71 |
| 91 | 46 | 56 | 65 | 72 |
| 92 | 47 | 57 | 66 | 73 |
| 93 | 48 | 59 | 68 | 75 |
| 94 | 50 | 60 | 69 | 76 |
| 95 | 52 | 63 | 72 | 78 |
| 96 | 53 | 65 | 74 | 79 |
| 97 | 56 | 66 | 75 | – |
| 98 | 59 | 68 | 77 | 80 |
| 99 | 64 | 72 | 81 | 84 |

**Table 31: Quantity Comparison—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K | | | Grade 1 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 1 | – | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 3 | 8 | 9 | 12 | 16 |
| 3 | 1 | 4 | 9 | 11 | 14 | 18 |
| 4 | 2 | 5 | 10 | 12 | 15 | 19 |
| 5 | – | 6 | 11 | 13 | 16 | 20 |
| 6 | 3 | – | – | 14 | 17 | 21 |
| 7 | – | 7 | 12 | 15 | 18 | 22 |
| 8 | 4 | 8 | 13 | – | 19 | 23 |
| 9 | 5 | – | – | 16 | 20 | 24 |
| 10 | – | 9 | 14 | – | – | – |
| 11 | 6 | 10 | 15 | 17 | 21 | 25 |
| 12 | – | – | – | – | 22 | 26 |
| 13 | – | – | – | 18 | – | – |
| 14 | 7 | 11 | 16 | – | 23 | 27 |
| 15 | – | – | – | 19 | – | – |
| 16 | – | – | – | – | – | – |
| 17 | 8 | 12 | 17 | – | 24 | 28 |
| 18 | – | – | – | 20 | – | – |
| 19 | – | – | – | – | – | – |
| 20 | 9 | 13 | 18 | – | 25 | 29 |
| 21 | – | – | – | 21 | – | – |
| 22 | – | – | – | – | – | – |
| 23 | – | 14 | 19 | – | 26 | 30 |
| 24 | – | – | – | – | – | – |
| 25 | 10 | – | – | 22 | – | – |

**Table 31: Quantity Comparison—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K | | | Grade 1 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 26 | – | – | – | – | – | – |
| 27 | – | 15 | 20 | – | 27 | 31 |
| 28 | – | – | – | 23 | – | – |
| 29 | 11 | – | – | – | – | – |
| 30 | – | 16 | 21 | – | 28 | 32 |
| 31 | – | – | – | – | – | – |
| 32 | – | – | – | 24 | – | – |
| 33 | – | – | – | – | – | – |
| 34 | 12 | 17 | 22 | – | 29 | 33 |
| 35 | – | – | – | – | – | – |
| 36 | – | – | – | – | – | – |
| 37 | – | – | – | 25 | – | – |
| 38 | 13 | 18 | 23 | – | – | – |
| 39 | – | – | – | – | 30 | 34 |
| 40 | – | – | – | – | – | – |
| 41 | – | – | – | – | – | – |
| 42 | – | – | – | 26 | – | – |
| 43 | 14 | 19 | 24 | – | 31 | 35 |
| 44 | – | – | – | – | – | – |
| 45 | – | – | – | 27 | – | – |
| 46 | – | – | – | – | – | – |
| 47 | 15 | 20 | 25 | – | – | – |
| 48 | – | – | – | – | 32 | 36 |
| 49 | – | – | – | 28 | – | – |
| 50 | – | – | – | – | – | – |

**Table 31: Quantity Comparison—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K | | | Grade 1 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 51 | 16 | 21 | 26 | – | – | – |
| 52 | – | – | – | – | 33 | 37 |
| 53 | – | – | – | – | – | – |
| 54 | – | – | – | 29 | – | – |
| 55 | 17 | – | – | – | – | – |
| 56 | – | – | – | – | – | – |
| 57 | – | 22 | 27 | – | – | – |
| 58 | – | – | – | – | 34 | 38 |
| 59 | 18 | – | – | – | – | – |
| 60 | – | – | – | – | – | – |
| 61 | – | 23 | 28 | 30 | – | – |
| 62 | – | – | – | – | 35 | 39 |
| 63 | 19 | – | – | – | – | – |
| 64 | – | – | – | 31 | – | – |
| 65 | – | 24 | 29 | – | – | – |
| 66 | – | – | – | – | 36 | 40 |
| 67 | 20 | – | – | – | – | – |
| 68 | – | – | – | 32 | – | – |
| 69 | – | 25 | 30 | – | 37 | 41 |
| 70 | – | – | – | – | – | – |
| 71 | 21 | – | – | 33 | – | – |
| 72 | – | 26 | 31 | – | – | – |
| 73 | – | – | – | – | 38 | 42 |
| 74 | – | – | – | – | – | – |
| 75 | 22 | – | – | 34 | – | – |

**Table 31: Quantity Comparison—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade K | | | Grade 1 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 76 | – | 27 | 32 | – | 39 | 43 |
| 77 | – | – | – | – | – | – |
| 78 | – | – | – | 35 | – | – |
| 79 | 23 | 28 | 33 | – | – | – |
| 80 | – | – | – | – | 40 | 44 |
| 81 | – | – | – | 36 | – | – |
| 82 | 24 | 29 | 34 | – | – | – |
| 83 | – | – | – | 37 | 41 | 45 |
| 84 | – | – | – | – | – | – |
| 85 | – | 30 | 35 | – | 42 | 46 |
| 86 | 25 | – | – | 38 | – | – |
| 87 | – | 31 | 36 | – | 43 | 47 |
| 88 | – | – | – | – | – | – |
| 89 | 26 | 32 | 37 | 39 | – | – |
| 90 | – | 33 | 38 | 40 | 44 | 48 |
| 91 | 27 | 34 | 39 | – | – | – |
| 92 | – | 35 | 40 | 41 | 45 | 49 |
| 93 | 28 | – | – | – | 46 | 50 |
| 94 | 29 | 36 | 41 | 42 | 47 | 51 |
| 95 | 30 | 38 | 43 | 43 | 48 | 52 |
| 96 | 31 | 39 | 44 | 44 | 49 | 53 |
| 97 | 33 | 41 | 46 | 45 | 50 | 54 |
| 98 | 35 | 44 | 49 | 48 | 51 | 55 |
| 99 | 37 | 47 | 52 | 51 | 54 | 58 |

**Table 32: Addition to 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 4 | 6 | 10 | 10 | 14 |
| 3 | 3 | 5 | 7 | 11 | 11 | 15 |
| 4 | – | – | – | – | – | – |
| 5 | 4 | 6 | 8 | 12 | 12 | 16 |
| 6 | 5 | – | – | 13 | 13 | 17 |
| 7 | – | 7 | 9 | 14 | 14 | 18 |
| 8 | 6 | – | – | – | – | – |
| 9 | – | – | – | 15 | 15 | 19 |
| 10 | 7 | 8 | 10 | 16 | 16 | 20 |
| 11 | – | – | – | – | 17 | 21 |
| 12 | – | 9 | 11 | 17 | 18 | 22 |
| 13 | 8 | – | – | – | – | – |
| 14 | – | – | – | – | 19 | 23 |
| 15 | – | – | – | 18 | – | – |
| 16 | 9 | 10 | 12 | – | – | – |
| 17 | – | – | – | – | – | – |
| 18 | – | – | – | 19 | – | – |
| 19 | – | 11 | 13 | – | 20 | 24 |
| 20 | 10 | – | – | – | – | – |
| 21 | – | – | – | 20 | – | – |
| 22 | – | – | – | – | – | – |
| 23 | – | 12 | 14 | – | – | – |
| 24 | 11 | – | – | 21 | 21 | 25 |
| 25 | – | – | – | – | – | – |

**Table 32: Addition to 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 26 | – | – | – | 22 | 22 | 26 |
| 27 | – | 13 | 15 | – | – | – |
| 28 | – | – | – | – | – | – |
| 29 | 12 | – | – | – | – | – |
| 30 | – | – | – | 23 | 23 | 27 |
| 31 | – | 14 | 16 | – | – | – |
| 32 | – | – | – | – | – | – |
| 33 | – | – | – | 24 | 24 | 28 |
| 34 | – | – | – | – | – | – |
| 35 | 13 | – | – | – | – | – |
| 36 | – | 15 | 17 | 25 | 25 | 29 |
| 37 | – | – | – | – | – | – |
| 38 | – | – | – | – | 26 | 30 |
| 39 | 14 | – | – | 26 | – | – |
| 40 | – | – | – | – | – | – |
| 41 | – | 16 | 18 | – | – | – |
| 42 | – | – | – | – | – | – |
| 43 | – | – | – | 27 | 27 | 31 |
| 44 | 15 | – | – | – | 28 | 32 |
| 45 | – | – | – | – | – | – |
| 46 | – | 17 | 19 | – | – | – |
| 47 | – | – | – | 28 | – | – |
| 48 | 16 | – | – | – | – | – |
| 49 | – | – | – | – | 29 | 33 |
| 50 | – | – | – | – | – | – |

**Table 32: Addition to 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 51 | – | 18 | 20 | 29 | – | – |
| 52 | 17 | – | – | – | 30 | 34 |
| 53 | – | – | – | – | – | – |
| 54 | – | 19 | 21 | 30 | – | – |
| 55 | – | – | – | – | – | – |
| 56 | – | – | – | – | 31 | 35 |
| 57 | 18 | – | – | 31 | 32 | 36 |
| 58 | – | – | – | – | 33 | 37 |
| 59 | – | 20 | 22 | – | – | – |
| 60 | – | – | – | 32 | – | – |
| 61 | 19 | – | – | – | 34 | 38 |
| 62 | – | – | – | – | 35 | 39 |
| 63 | – | 21 | 23 | – | – | – |
| 64 | – | – | – | 33 | – | – |
| 65 | 20 | – | – | – | 36 | 40 |
| 66 | – | 22 | 24 | 34 | 37 | 41 |
| 67 | – | – | – | – | – | – |
| 68 | – | – | – | – | 40 | 44 |
| 69 | 21 | 23 | 25 | – | – | – |
| 70 | – | – | – | 35 | – | – |
| 71 | – | – | – | – | 41 | 45 |
| 72 | – | – | – | – | 42 | 46 |
| 73 | 22 | 24 | 26 | – | – | – |
| 74 | – | – | – | 36 | – | – |
| 75 | – | – | – | – | 44 | 48 |

**Table 32: Addition to 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 76 | 23 | 25 | 27 | 37 | 45 | 49 |
| 77 | — | — | — | — | 46 | 50 |
| 78 | — | — | — | — | 47 | 51 |
| 79 | — | 26 | 28 | 38 | — | — |
| 80 | 24 | — | — | — | — | — |
| 81 | — | — | — | 39 | — | — |
| 82 | 25 | 27 | 29 | — | 48 | 52 |
| 83 | — | — | — | 40 | 49 | 53 |
| 84 | — | 28 | 30 | — | 51 | 55 |
| 85 | 26 | 29 | 31 | — | 53 | 57 |
| 86 | — | — | — | 41 | 56 | 60 |
| 87 | 27 | 30 | 32 | 42 | 57 | 61 |
| 88 | — | — | — | — | — | — |
| 89 | 28 | 31 | 33 | 43 | 60 | 64 |
| 90 | 29 | 32 | 34 | 44 | 63 | 67 |
| 91 | — | — | — | 45 | — | — |
| 92 | 30 | 33 | 35 | — | 64 | 68 |
| 93 | 31 | 34 | 36 | 46 | — | — |
| 94 | — | 35 | 37 | 47 | 66 | 70 |
| 95 | 33 | 36 | 38 | 48 | — | — |
| 96 | 35 | 38 | 40 | 49 | 67 | 71 |
| 97 | 37 | 39 | 41 | 51 | 68 | 72 |
| 98 | 39 | 41 | 43 | 53 | 77 | 81 |
| 99 | 42 | 47 | 49 | 58 | 80 | 84 |

**Table 33: Addition to 20—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 2 Fall | Grade 2 Winter | Grade 2 Spring |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 3 | 4 | 9 |
| 3 | 4 | 5 | 10 |
| 4 | — | — | — |
| 5 | 5 | 6 | 11 |
| 6 | — | — | — |
| 7 | 6 | 7 | 12 |
| 8 | — | 8 | 13 |
| 9 | 7 | — | — |
| 10 | — | — | — |
| 11 | 8 | 9 | 14 |
| 12 | — | — | — |
| 13 | — | — | — |
| 14 | 9 | 10 | 15 |
| 15 | — | — | — |
| 16 | — | 11 | 16 |
| 17 | — | — | — |
| 18 | 10 | — | — |
| 19 | — | 12 | 17 |
| 20 | — | — | — |
| 21 | 11 | — | — |
| 22 | — | 13 | 18 |
| 23 | — | — | — |
| 24 | — | — | — |
| 25 | 12 | 14 | 19 |
| 26 | — | — | — |

**Table 33: Addition to 20—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 2 Fall | Grade 2 Winter | Grade 2 Spring |
|---|---|---|---|
| 27 | — | — | — |
| 28 | — | — | — |
| 29 | 13 | 15 | 20 |
| 30 | — | — | — |
| 31 | — | — | — |
| 32 | — | — | — |
| 33 | — | 16 | 21 |
| 34 | 14 | — | — |
| 35 | — | — | — |
| 36 | — | — | — |
| 37 | — | 17 | 22 |
| 38 | 15 | — | — |
| 39 | — | — | — |
| 40 | — | — | — |
| 41 | — | — | — |
| 42 | — | 18 | 23 |
| 43 | — | — | — |
| 44 | 16 | — | — |
| 45 | — | — | — |
| 46 | — | — | — |
| 47 | — | 19 | 24 |
| 48 | 17 | — | — |
| 49 | — | — | — |
| 50 | — | — | — |
| 51 | — | — | — |
| 52 | — | — | — |

**Table 33: Addition to 20—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 2 Fall | Grade 2 Winter | Grade 2 Spring |
|---|---|---|---|
| 53 | – | 20 | 25 |
| 54 | 18 | – | – |
| 55 | – | – | – |
| 56 | – | 21 | 26 |
| 57 | – | – | – |
| 58 | – | – | – |
| 59 | 19 | – | – |
| 60 | – | 22 | 27 |
| 61 | – | – | – |
| 62 | – | – | – |
| 63 | – | – | – |
| 64 | 20 | 23 | 28 |
| 65 | – | – | – |
| 66 | – | – | – |
| 67 | – | 24 | 29 |
| 68 | – | – | – |
| 69 | 21 | – | – |
| 70 | – | 25 | 30 |
| 71 | – | – | – |
| 72 | – | – | – |
| 73 | 22 | 26 | 31 |
| 74 | – | – | – |
| 75 | – | 27 | 32 |
| 76 | – | – | – |
| 77 | 23 | – | – |
| 78 | – | 28 | 33 |

**Table 33: Addition to 20—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 2 Fall | Grade 2 Winter | Grade 2 Spring |
|---|---|---|---|
| 79 | — | — | — |
| 80 | — | 29 | 34 |
| 81 | 24 | — | — |
| 82 | — | 30 | 35 |
| 83 | — | 31 | 36 |
| 84 | 25 | — | — |
| 85 | — | 32 | 37 |
| 86 | 26 | 33 | 38 |
| 87 | 27 | 34 | 39 |
| 88 | — | — | — |
| 89 | 28 | 35 | 40 |
| 90 | 29 | 36 | 41 |
| 91 | — | 37 | 42 |
| 92 | 30 | 40 | 45 |
| 93 | 31 | 44 | 49 |
| 94 | 32 | 48 | 53 |
| 95 | 34 | 58 | 63 |
| 96 | 35 | 67 | 72 |
| 97 | 37 | 73 | 78 |
| 98 | 39 | 76 | 81 |
| 99 | 46 | 79 | 84 |

**Table 34: Subtraction from 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 2 | 4 | 9 | 4 | 7 | 13 |
| 3 | 3 | – | – | 5 | 8 | 14 |
| 4 | – | 5 | 10 | – | – | – |
| 5 | 4 | – | – | 6 | 9 | 15 |
| 6 | – | 6 | 11 | 7 | – | – |
| 7 | 5 | – | – | 8 | 10 | 16 |
| 8 | – | 7 | 12 | – | – | – |
| 9 | 6 | – | – | 9 | – | – |
| 10 | – | 8 | 13 | – | 11 | 17 |
| 11 | – | – | – | 10 | – | – |
| 12 | 7 | 9 | 14 | – | – | – |
| 13 | – | – | – | – | – | – |
| 14 | – | – | – | 11 | – | – |
| 15 | – | 10 | 15 | – | 12 | 18 |
| 16 | 8 | – | – | – | 13 | 19 |
| 17 | – | 11 | 16 | 12 | – | – |
| 18 | – | – | – | – | – | – |
| 19 | – | – | – | – | – | – |
| 20 | 9 | – | – | 13 | – | – |
| 21 | – | 12 | 17 | – | – | – |
| 22 | – | – | – | – | 16 | 22 |
| 23 | 10 | – | – | – | 17 | 23 |
| 24 | – | – | – | 14 | – | – |
| 25 | – | – | – | – | – | – |

**Table 34: Subtraction from 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 26 | – | 13 | 18 | – | – | – |
| 27 | – | – | – | – | 18 | 24 |
| 28 | – | – | – | 15 | 19 | 25 |
| 29 | 11 | – | – | – | – | – |
| 30 | – | 14 | 19 | – | – | – |
| 31 | – | – | – | 16 | – | – |
| 32 | – | – | – | – | – | – |
| 33 | – | – | – | – | – | – |
| 34 | – | 15 | 20 | – | – | – |
| 35 | 12 | – | – | – | – | – |
| 36 | – | – | – | 17 | 21 | 27 |
| 37 | – | – | – | – | – | – |
| 38 | – | 16 | 21 | – | – | – |
| 39 | – | – | – | 18 | – | – |
| 40 | – | – | – | – | 22 | 28 |
| 41 | 13 | – | – | – | – | – |
| 42 | – | – | – | – | – | – |
| 43 | – | – | – | 19 | 23 | 29 |
| 44 | – | 17 | 22 | – | – | – |
| 45 | – | – | – | – | – | – |
| 46 | – | – | – | – | – | – |
| 47 | 14 | – | – | 20 | 24 | 30 |
| 48 | – | – | – | – | – | – |
| 49 | – | 18 | 23 | – | – | – |
| 50 | – | – | – | – | – | – |

**Table 34: Subtraction from 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 51 | – | – | – | 21 | – | – |
| 52 | 15 | – | – | – | 25 | 31 |
| 53 | – | 19 | 24 | – | 26 | 32 |
| 54 | – | – | – | 22 | – | – |
| 55 | – | – | – | – | 27 | 33 |
| 56 | 16 | – | – | – | – | – |
| 57 | – | – | – | – | – | – |
| 58 | – | 20 | 25 | 23 | – | – |
| 59 | – | – | – | – | – | – |
| 60 | – | – | – | – | – | – |
| 61 | 17 | 21 | 26 | – | 28 | 34 |
| 62 | – | – | – | 24 | – | – |
| 63 | – | – | – | – | 29 | 35 |
| 64 | – | 22 | 27 | – | – | – |
| 65 | – | – | – | – | 30 | 36 |
| 66 | – | – | – | 25 | 31 | 37 |
| 67 | 18 | 23 | 28 | – | – | – |
| 68 | – | – | – | – | – | – |
| 69 | – | – | – | 26 | 32 | 38 |
| 70 | – | – | – | – | 33 | 39 |
| 71 | 19 | 24 | 29 | – | 34 | 40 |
| 72 | – | – | – | 27 | – | – |
| 73 | – | – | – | – | – | – |
| 74 | 20 | 25 | 30 | – | – | – |
| 75 | – | – | – | 28 | 36 | 42 |

**Table 34: Subtraction from 10—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 1 | | | Grade 2 | | |
|---|---|---|---|---|---|---|
| | Fall | Winter | Spring | Fall | Winter | Spring |
| 76 | – | – | – | – | 37 | 43 |
| 77 | – | 26 | 31 | – | 38 | 44 |
| 78 | 21 | – | – | 29 | – | – |
| 79 | – | – | – | – | – | – |
| 80 | 22 | 27 | 32 | 30 | 39 | 45 |
| 81 | – | – | – | – | 41 | 47 |
| 82 | – | 28 | 33 | 31 | 43 | 49 |
| 83 | 23 | 29 | 34 | – | – | – |
| 84 | – | 30 | 35 | – | – | – |
| 85 | – | – | – | 32 | 44 | 50 |
| 86 | 24 | 31 | 36 | – | – | – |
| 87 | – | – | – | 33 | 45 | 51 |
| 88 | 25 | 32 | 37 | 34 | – | – |
| 89 | 26 | 33 | 38 | – | 46 | 52 |
| 90 | – | 35 | 40 | 35 | – | – |
| 91 | 27 | 36 | 41 | 36 | – | – |
| 92 | 28 | 37 | 42 | – | 49 | 55 |
| 93 | 29 | 40 | 45 | 37 | 50 | 56 |
| 94 | 30 | 44 | 49 | 38 | 53 | 59 |
| 95 | 31 | 52 | 57 | 40 | 54 | 60 |
| 96 | 33 | 63 | 68 | 41 | 55 | 61 |
| 97 | 35 | 67 | 72 | 44 | 60 | 66 |
| 98 | 37 | 70 | 75 | 46 | – | – |
| 99 | 44 | 75 | 80 | 54 | 62 | 68 |

**Table 35: Mixed Addition and Subtraction—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 2 | 1 | 2 | 4 |
| 3 | 2 | 3 | 5 |
| 4 | – | – | – |
| 5 | 3 | 4 | 6 |
| 6 | – | – | – |
| 7 | – | – | – |
| 8 | 4 | 5 | 7 |
| 9 | – | – | – |
| 10 | – | – | – |
| 11 | 5 | – | – |
| 12 | – | 6 | 8 |
| 13 | – | – | – |
| 14 | 6 | – | – |
| 15 | – | – | – |
| 16 | – | – | – |
| 17 | – | – | – |
| 18 | – | 7 | 9 |
| 19 | – | – | – |
| 20 | – | – | – |
| 21 | 7 | – | – |
| 22 | – | 8 | 10 |
| 23 | – | – | – |
| 24 | – | – | – |
| 25 | – | – | – |
| 26 | 8 | – | – |

**Table 35: Mixed Addition and Subtraction—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 27 | – | 9 | 11 |
| 28 | – | – | – |
| 29 | – | – | – |
| 30 | – | – | – |
| 31 | 9 | – | – |
| 32 | – | 10 | 12 |
| 33 | – | – | – |
| 34 | – | – | – |
| 35 | – | – | – |
| 36 | – | – | – |
| 37 | 10 | 11 | 13 |
| 38 | – | – | – |
| 39 | – | – | – |
| 40 | – | – | – |
| 41 | – | – | – |
| 42 | – | – | – |
| 43 | 11 | – | – |
| 44 | – | 12 | 14 |
| 45 | – | – | – |
| 46 | – | – | – |
| 47 | – | – | – |
| 48 | – | – | – |
| 49 | – | – | – |
| 50 | 12 | 13 | 15 |
| 51 | – | – | – |
| 52 | – | – | – |

**Table 35: Mixed Addition and Subtraction—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 53 | – | – | – |
| 54 | – | 14 | 16 |
| 55 | – | – | – |
| 56 | 13 | – | – |
| 57 | – | – | – |
| 58 | – | 15 | 17 |
| 59 | – | – | – |
| 60 | – | – | – |
| 61 | 14 | – | – |
| 62 | – | 16 | 18 |
| 63 | – | – | – |
| 64 | – | – | – |
| 65 | – | – | – |
| 66 | 15 | 17 | 19 |
| 67 | – | – | – |
| 68 | – | – | – |
| 69 | – | – | – |
| 70 | – | – | – |
| 71 | – | 18 | 20 |
| 72 | 16 | – | – |
| 73 | – | – | – |
| 74 | – | 19 | 21 |
| 75 | – | – | – |
| 76 | 17 | – | – |
| 77 | – | – | – |
| 78 | – | 20 | 22 |

**Table 35: Mixed Addition and Subtraction—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 79 | – | – | – |
| 80 | 18 | – | – |
| 81 | – | 21 | 23 |
| 82 | – | – | – |
| 83 | 19 | 22 | 24 |
| 84 | – | – | – |
| 85 | 20 | 23 | 25 |
| 86 | – | – | – |
| 87 | – | 24 | 26 |
| 88 | 21 | 25 | 27 |
| 89 | 22 | – | – |
| 90 | 23 | 26 | 28 |
| 91 | – | 27 | 29 |
| 92 | 24 | – | – |
| 93 | 25 | 28 | 30 |
| 94 | – | 29 | 31 |
| 95 | 26 | 30 | 32 |
| 96 | 28 | 32 | 34 |
| 97 | 29 | 34 | 36 |
| 98 | 31 | 36 | 38 |
| 99 | 36 | 39 | 41 |

**Table 36: Multiplication to 100—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 1 | – | 0 | 0 |
| 2 | – | 2 | 8 |
| 3 | – | – | – |
| 4 | 0 | 3 | 9 |
| 5 | – | – | – |
| 6 | – | 4 | 10 |
| 7 | 1 | – | – |
| 8 | – | 5 | 11 |
| 9 | – | – | – |
| 10 | 2 | – | – |
| 11 | – | 6 | 12 |
| 12 | – | – | – |
| 13 | 3 | 7 | 13 |
| 14 | – | – | – |
| 15 | – | 8 | 14 |
| 16 | – | – | – |
| 17 | 4 | – | – |
| 18 | – | 9 | 15 |
| 19 | – | – | – |
| 20 | – | – | – |
| 21 | 5 | 10 | 16 |
| 22 | – | – | – |
| 23 | – | – | – |
| 24 | – | – | – |
| 25 | 6 | 11 | 17 |
| 26 | – | – | – |

**Table 36: Multiplication to 100—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 27 | – | 12 | 18 |
| 28 | – | – | – |
| 29 | – | – | – |
| 30 | 7 | – | – |
| 31 | – | – | – |
| 32 | – | 13 | 19 |
| 33 | 8 | – | – |
| 34 | – | – | – |
| 35 | – | – | – |
| 36 | – | 14 | 20 |
| 37 | – | – | – |
| 38 | 9 | – | – |
| 39 | – | – | – |
| 40 | – | – | – |
| 41 | – | 15 | 21 |
| 42 | 10 | – | – |
| 43 | – | – | – |
| 44 | – | 16 | 22 |
| 45 | – | – | – |
| 46 | – | – | – |
| 47 | 11 | – | – |
| 48 | – | 17 | 23 |
| 49 | – | – | – |
| 50 | 12 | – | – |
| 51 | – | – | – |
| 52 | – | 18 | 24 |

**Table 36: Multiplication to 100—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 53 | – | – | – |
| 54 | 13 | – | – |
| 55 | – | – | – |
| 56 | – | 19 | 25 |
| 57 | – | – | – |
| 58 | – | – | – |
| 59 | 14 | – | – |
| 60 | – | 20 | 26 |
| 61 | – | – | – |
| 62 | 15 | – | – |
| 63 | – | 21 | 27 |
| 64 | – | – | – |
| 65 | – | – | – |
| 66 | 16 | 22 | 28 |
| 67 | – | – | – |
| 68 | – | – | – |
| 69 | – | – | – |
| 70 | 17 | 23 | 29 |
| 71 | – | – | – |
| 72 | – | – | – |
| 73 | 18 | 24 | 30 |
| 74 | – | – | – |
| 75 | 19 | 25 | 31 |
| 76 | – | – | – |
| 77 | – | 26 | 32 |
| 78 | 20 | – | – |

**Table 36: Multiplication to 100—Percentile Rank to Correct Per Minute Conversions**

| Percentile Rank (PR) | Grade 3 Fall | Grade 3 Winter | Grade 3 Spring |
|---|---|---|---|
| 79 | – | 27 | 33 |
| 80 | – | – | – |
| 81 | 21 | 28 | 34 |
| 82 | – | – | – |
| 83 | 22 | – | – |
| 84 | 23 | 29 | 35 |
| 85 | – | 30 | 36 |
| 86 | 24 | – | – |
| 87 | – | 31 | 37 |
| 88 | 25 | 33 | 39 |
| 89 | 26 | – | – |
| 90 | 27 | 34 | 40 |
| 91 | 28 | 35 | 41 |
| 92 | 29 | 37 | 43 |
| 93 | 30 | 39 | 45 |
| 94 | 31 | 40 | 46 |
| 95 | 32 | 41 | 47 |
| 96 | 35 | 43 | 49 |
| 97 | 38 | 45 | 51 |
| 98 | 42 | 49 | 55 |
| 99 | 50 | 53 | 59 |

# Conversions Between Normal Curve Equivalents (NCE) and Percentile Ranks (PR)

**Table 37: Normal Curve Equivalent to Percentile Rank Conversions**

| Normal Curve Equivalent Range | | Percentile Rank |
|---|---|---|
| Low | High | |
| 1.0 | 4.0 | 1 |
| 4.1 | 8.5 | 2 |
| 8.6 | 11.7 | 3 |
| 11.8 | 14.1 | 4 |
| 14.2 | 16.2 | 5 |
| 16.3 | 18.0 | 6 |
| 18.1 | 19.6 | 7 |
| 19.7 | 21.0 | 8 |
| 21.1 | 22.3 | 9 |
| 22.4 | 23.5 | 10 |
| 23.6 | 24.6 | 11 |
| 24.7 | 25.7 | 12 |
| 25.8 | 26.7 | 13 |
| 26.8 | 27.6 | 14 |
| 27.7 | 28.5 | 15 |
| 28.6 | 29.4 | 16 |
| 29.5 | 30.2 | 17 |
| 30.3 | 31.0 | 18 |
| 31.1 | 31.8 | 19 |
| 31.9 | 32.6 | 20 |
| 32.7 | 33.3 | 21 |
| 33.4 | 34.0 | 22 |
| 34.1 | 34.7 | 23 |
| 34.8 | 35.4 | 24 |
| 35.5 | 36.0 | 25 |

**Table 37: Normal Curve Equivalent to Percentile Rank Conversions**

| Normal Curve Equivalent Range | | Percentile Rank |
|---|---|---|
| Low | High | |
| 36.1 | 36.7 | 26 |
| 36.8 | 37.3 | 27 |
| 37.4 | 38.0 | 28 |
| 38.1 | 38.6 | 29 |
| 38.7 | 39.2 | 30 |
| 39.3 | 39.8 | 31 |
| 39.9 | 40.4 | 32 |
| 40.5 | 40.9 | 33 |
| 41.0 | 41.5 | 34 |
| 41.6 | 42.1 | 35 |
| 42.2 | 42.7 | 36 |
| 42.8 | 43.2 | 37 |
| 43.3 | 43.8 | 38 |
| 43.9 | 44.3 | 39 |
| 44.4 | 44.9 | 40 |
| 45.0 | 45.4 | 41 |
| 45.5 | 45.9 | 42 |
| 46.0 | 46.5 | 43 |
| 46.6 | 47.0 | 44 |
| 47.1 | 47.5 | 45 |
| 47.6 | 48.1 | 46 |
| 48.2 | 48.6 | 47 |
| 48.7 | 49.1 | 48 |
| 49.2 | 49.7 | 49 |
| 49.8 | 50.2 | 50 |
| 50.3 | 50.7 | 51 |
| 50.8 | 51.2 | 52 |

**Table 37: Normal Curve Equivalent to Percentile Rank Conversions**

| Normal Curve Equivalent Range | | Percentile Rank |
|---|---|---|
| **Low** | **High** | |
| 51.3 | 51.8 | 53 |
| 51.9 | 52.3 | 54 |
| 52.4 | 52.8 | 55 |
| 52.9 | 53.4 | 56 |
| 53.5 | 53.9 | 57 |
| 54.0 | 54.4 | 58 |
| 54.5 | 55.0 | 59 |
| 55.1 | 55.5 | 60 |
| 55.6 | 56.1 | 61 |
| 56.2 | 56.6 | 62 |
| 56.7 | 57.2 | 63 |
| 57.3 | 57.8 | 64 |
| 57.9 | 58.3 | 65 |
| 58.4 | 58.9 | 66 |
| 59.0 | 59.5 | 67 |
| 59.6 | 60.1 | 68 |
| 60.2 | 60.7 | 69 |
| 60.8 | 61.3 | 70 |
| 61.4 | 61.9 | 71 |
| 62.0 | 62.5 | 72 |
| 62.6 | 63.1 | 73 |
| 63.2 | 63.8 | 74 |
| 63.9 | 64.5 | 75 |
| 64.6 | 65.1 | 76 |
| 65.2 | 65.8 | 77 |
| 65.9 | 66.5 | 78 |
| 66.6 | 67.3 | 79 |

**Table 37: Normal Curve Equivalent to Percentile Rank Conversions**

| Normal Curve Equivalent Range | | Percentile Rank |
|---|---|---|
| Low | High | |
| 67.4 | 68.0 | 80 |
| 68.1 | 68.6 | 81 |
| 68.7 | 69.6 | 82 |
| 69.7 | 70.4 | 83 |
| 70.5 | 71.3 | 84 |
| 71.4 | 72.2 | 85 |
| 72.3 | 73.1 | 86 |
| 73.2 | 74.1 | 87 |
| 74.2 | 75.2 | 88 |
| 75.3 | 76.3 | 89 |
| 76.4 | 77.5 | 90 |
| 77.6 | 78.8 | 91 |
| 78.9 | 80.2 | 92 |
| 80.3 | 81.7 | 93 |
| 81.8 | 83.5 | 94 |
| 83.6 | 85.5 | 95 |
| 85.6 | 88.0 | 96 |
| 88.1 | 91.0 | 97 |
| 91.1 | 95.4 | 98 |
| 95.5 | 99.0 | 99 |

**Table 38: Percentile Rank to Normal Curve Equivalent**

| Percentile Rank | Normal Curve Equivalent |
|---|---|
| 1 | 1.0 |
| 2 | 6.7 |
| 3 | 10.4 |
| 4 | 13.1 |
| 5 | 15.4 |

**Table 38: Percentile Rank to Normal Curve Equivalent**

| Percentile Rank | Normal Curve Equivalent |
|---|---|
| 6 | 17.3 |
| 7 | 18.9 |
| 8 | 20.4 |
| 9 | 21.8 |
| 10 | 23.0 |
| 11 | 24.2 |
| 12 | 25.3 |
| 13 | 26.3 |
| 14 | 27.2 |
| 15 | 28.2 |
| 16 | 29.1 |
| 17 | 29.9 |
| 18 | 30.7 |
| 19 | 31.5 |
| 20 | 32.3 |
| 21 | 33.0 |
| 22 | 33.7 |
| 23 | 34.4 |
| 24 | 35.1 |
| 25 | 35.8 |
| 26 | 36.5 |
| 27 | 37.1 |
| 28 | 37.7 |
| 29 | 38.3 |
| 30 | 39.0 |
| 31 | 39.6 |
| 32 | 40.1 |
| 33 | 40.7 |

**Table 38: Percentile Rank to Normal Curve Equivalent**

| Percentile Rank | Normal Curve Equivalent |
|---|---|
| 34 | 41.3 |
| 35 | 41.9 |
| 36 | 42.5 |
| 37 | 43.0 |
| 38 | 43.6 |
| 39 | 44.1 |
| 40 | 44.7 |
| 41 | 45.2 |
| 42 | 45.8 |
| 43 | 46.3 |
| 44 | 46.8 |
| 45 | 47.4 |
| 46 | 47.9 |
| 47 | 48.4 |
| 48 | 48.9 |
| 49 | 49.5 |
| 50 | 50.0 |
| 51 | 50.5 |
| 52 | 51.1 |
| 53 | 51.6 |
| 54 | 52.1 |
| 55 | 52.6 |
| 56 | 53.2 |
| 57 | 53.7 |
| 58 | 54.2 |
| 59 | 54.8 |
| 60 | 55.3 |
| 61 | 55.9 |

**Table 38: Percentile Rank to Normal Curve Equivalent**

| Percentile Rank | Normal Curve Equivalent |
|---|---|
| 62 | 56.4 |
| 63 | 57.0 |
| 64 | 57.5 |
| 65 | 58.1 |
| 66 | 58.7 |
| 67 | 59.3 |
| 68 | 59.9 |
| 69 | 60.4 |
| 70 | 61.0 |
| 71 | 61.7 |
| 72 | 62.3 |
| 73 | 62.9 |
| 74 | 63.5 |
| 75 | 64.2 |
| 76 | 64.9 |
| 77 | 65.6 |
| 78 | 66.3 |
| 79 | 67.0 |
| 80 | 67.7 |
| 81 | 68.5 |
| 82 | 69.3 |
| 83 | 70.1 |
| 84 | 70.9 |
| 85 | 71.8 |
| 86 | 72.8 |
| 87 | 73.7 |
| 88 | 74.7 |
| 89 | 75.8 |

**Table 38:  Percentile Rank to Normal Curve Equivalent**

| Percentile Rank | Normal Curve Equivalent |
|---|---|
| 90 | 77.0 |
| 91 | 78.2 |
| 92 | 79.6 |
| 93 | 81.1 |
| 94 | 82.7 |
| 95 | 84.6 |
| 96 | 86.9 |
| 97 | 89.6 |
| 98 | 93.3 |
| 99 | 99.0 |

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